# D-Cluster Dynamics and Fusion Rate by Langevin Equation

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## ABSTRACT

Condensed matter nuclear effect, especially 4D-cluster fusion, in metal-deuterium complex systems, has been studied by applying Langevin equations.

One dimensional Langevin equations for solving time-dependent d-d distance $R_{dd}(t)$ for deuteron-clusters under the Platonic symmetry were formulated for D-atom, $D_2$ molecule, $D_2^+$ ion, $D_3^+$ ion, 4D/TSC and $6D^{2-}$/OSC. Established values of ground state d-d distances $R_{gs}$ were reproduced by expectation-value equations, which were obtained by ensemble averaging with weight of quantum mechanical wave functions (Gaussian wave functions), for D-atom, $D_2$, $D_2^+$, and $D_3^+$ molecule.

In analogy to the above Langevin equations, the Langevin equation for 4D/TSC under the tetrahedral double Platonic symmetry was derived and numerically solved by the Verlet time-step method. It was shown that only 4D/TSC among 5 D-systems except D-atom could condense ultimately from $R_{dd}(t=0)$=74 pm to very small charge neutral entity with about 20 fm radius at TSC-min state after about 1.4 fs condensation time. The $6D^{2-}$/OSC system converged at $R_{gs}$=about 40 pm, namely converged on the way of condensation from $R_{dd}(t=0)$=74 pm.

Time-dependent Coulomb barrier penetration probabilities (barrier factors) for condensing 4D/TSC were calculated by the Heavy Mass Electronic Quasi-Particle Expansion Method (HMEQPET). 4D fusion rate per TSC generation was obtained based on the Fermi's first golden rule to result in almost 100% 4D fusion reaction per 4D/TSC generation. Fusion rates were compared with those of muonic dd molecule, $D_2$ molecule and dde*(2,2) Cooper pair molecule to meet good consistency. Major nuclear products of 4D fusion are two 23.8 MeV α-particles. 4H/TSC should condense in the same way until when TSC-min state with classical electron radius (2.8 fm) comes, but no strong interaction exists among protons and will make 1p to 4p capture transmutations with host metal nuclei when 4H/TSC has sufficient drift (CMS) momentum.

**Keywords:** D-cluster fusion, dynamics, condensation, 4D/TSC, D-molecules, Langevin equation, barrier factors, 4D fusion, helium-4

## 1. Introduction

To explain apparent hard-radiation-less excess heat with helium-4 ash in CMNS (condensed matter nuclear science) experiments, especially in dynamic PdDx systems, we have done a long series of study for modeling D-cluster (or multi-body deuteron) fusion reaction mechanisms to reach at our latest theory based on Langevin equations[1,2].

This paper describes the basics of formulation of Langevin equations for D-cluster dynamics, especially for D-atom, $D_2$ molecule, $D_2^+$ ion, $D_3^+$ ion, 4D/TSC (tetrahedral symmetric condensate) and $6D^{2-}$/OSC (octahedral symmetric condensate).

First one-dimensional Langevin equations for D-clusters with the $R_{dd}$ d-d distance are formulated under the Platonic symmtery[1,3] of multi-particle D-cluster systems with deuterons and quantum-mechanical electron centers. Under the orthogonally coupled Platonic symmetry for a Platonic deuteron-system and a Platonic electron system, dynamic equations for so-many-body system of deuterons and electrons with metal atoms (more than 4 deuterons plus 4 1s electrons of deuterium atoms plus 40 4d-shell electrons of 4 Pd atoms in fcc lattice plus surrounding lattice atoms under D-phonon excited states should be considered in our modeling[7]), a simple one-dimensional Langevin equation for the inter-nuclear d-d distance $R_{dd}$ can be formulated, as we will show in this paper. By the ensemble averaging of one-dimensional Langevin equation with the weight of quantum mechanical wave-functions for electrons and deuterons, we can further derive a time dependent equation for expectation value $<R_{dd}>$, which is nonlinear, but can be solved by the Verlet time step method[2]. We show in this paper that only 4D(orH)/TSC can condense ultimately to be finally very small charge neutral entity with about 10-20 fm radius. At the final stage of 4D/TSC condensation in about $2x10^{-20}$ s, 4D fusion with 2 $^4$He products takes place with almost 100% probability, according to our HMEQPET calculation for barrier factors and fusion rate formula by the Fermi's first golden rule[2].

In the next section, we show the derivation of Langevin equations for known systems as D-atom, $D_2$ molecule, $D_2^+$ ion, and $D_3^+$ ion. This procedure gives the basics for formulating Langevin equations of complex D-cluster systems as 4D/TSC and 6D/OSC. In analogy, we apply the methodology and derive Langevin equations for 4D/TSC and 6D/OSC condensation motions in the following section.

## 2. Langevin Equations for Known D-Systems

### 2.1. Langevin equation in general

The Langevin equation is useful to treat dynamic motion of particles under friction (or constraint) and random fluctuation of force-field.

$$m\frac{d^2R}{dt^2} = -F_c - \varsigma\frac{dR}{dt} + f'(t) \tag{1}$$

Here m is the particle mass, R is particle position, $F_c$ is the Coulombic force, $\varsigma$ is the coefficient for friction (or constraint) and f'(t) is the randomly fluctuated force term (white noise), for our deuterons plus electrons system.

### 2.2. Langevin equation for D-atom

In **Fig. 1**, simple quantum mechanical image of D-atom is drawn. In the view of Platonic symmetry, D-atom is the orthogonal coupling of central point (deuteron) and sphere (electron-wave). The Langevin equation is given as balance of the centripetal force of Coulombic

attraction between plus-charged deuteron and minus-charged electron and the centrifugal force of electron rotation around central point (deuteron);

$$m_e \frac{d^2 R_{de}}{dt^2} = -\frac{e^2}{[R_{de}]^2} + \frac{m_e v_e^2}{R_{de}} + f(t) \tag{2}$$

Here, $m_e$ is the electron mass, $R_{de}$ is the d-e distance, e is the unit charge and $v_e$ is

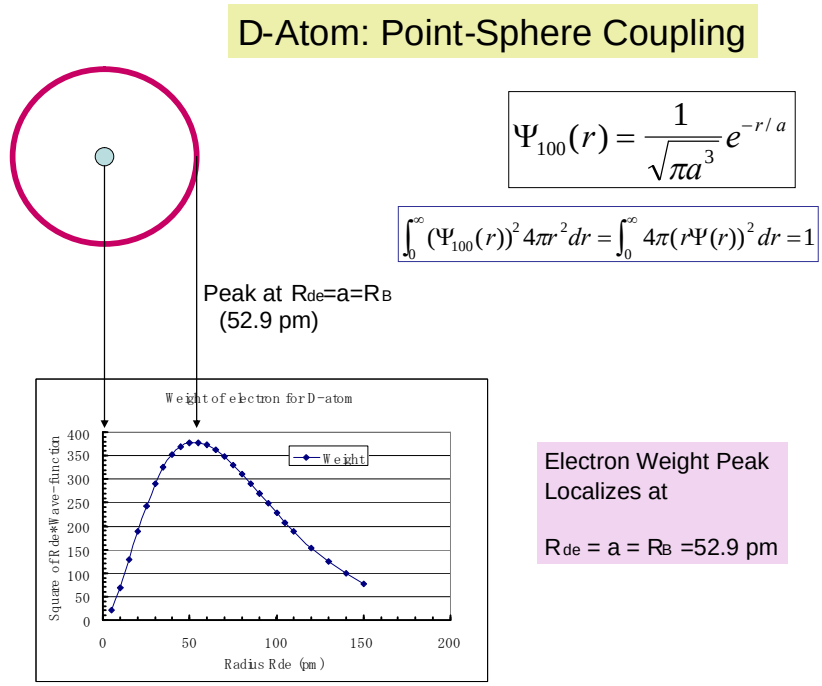



Figure. 1. Quantum mechanical image of D-atom

the electron velocity. We have no friction in this case. By taking ensemble average of Eq. (2) with the weight of squared 1-s wave function $\Psi_{100}{}^2$, we obtain,

$$m_e \frac{d^2 \langle R_{de} \rangle}{dt^2} = -\left\langle \frac{e^2}{R_{de}^2} \right\rangle + \left\langle \frac{m_e v_e^2}{R_{de}} \right\rangle = 0 \tag{3}$$

The right side becomes zero, because of the average kinetic energy

$\langle E_{KE} \rangle = \frac{1}{2} m_e \langle v_e^2 \rangle = \frac{e^2}{2R_B} = 13.6eV$ and the average Coulomb energy

$\langle E_C \rangle = -\frac{e^2}{R_B} = -27.2eV$ as well known for hydrogen (or deuterium) atom.

We can integrate Eq. (3) over time to get,

$$m_e \frac{d\langle R_{de}\rangle}{dt} = F(T) = \int_0^T f(t)dt = \langle f(t)\rangle = 0 \tag{4}$$

The time-average (integral) of random fluctuation f(t) is equal to the ensemble average <f(t)> due to the ergodic process. We integrate Eq. (4) to obtain the well known result as,

$$\langle R_{de}\rangle(t) = R_0 = R_B = 52.9pm \tag{5}$$

Namely, expectation value $<R_{de}>$ of radial electron orbit is constant to be Bohr radius $R_B$ = 52.9 pm.

### 2.3. Quantum Mechanical Ensemble Average for D-Cluster

Since both positions of electrons and deuterons fluctuate quantum mechanically for D-cluster systems, we need to average with both weights of wave functions for electrons and deuterons. Applying the Born-Oppenheimer (adiabatic) approximation for total wave function, we can make step-wise averaging for electron-waves and then for deuteron-waves. The adiabatic wave function for $D_2$ molecule is;

$$\Psi(R_{dd}; r_{A1}, r_{A2}, r_{B1}, r_{B2}) = \Psi_{2D} \cdot X(R_{dd}) \tag{6}$$

The electron wave function of $D_2$ molecule is given[4] by,

$$\Psi_{2D} = \frac{1}{\sqrt{(2+2\Delta)}}\left[\Psi_{100}(r_{A1})\Psi_{100}(r_{B2}) + \Psi_{100}(r_{A2})\Psi_{100}(r_{B1})\right]X_s(S1, S2) \tag{7}$$

And the wave function for a d-d pair is approximated by the Gaussian wave function as, rewriting X with $\Psi$ and putting $R_{dd}$ = R,

$$\Psi(R, R') = \frac{1}{\sqrt{2\pi\sigma^2}}\exp(-(R'-R)^2/(2\sigma^2)) \tag{8}$$

Quantum-mechanical ensemble average of observable G is given by,

$$\langle G\rangle_{ensemble} = \langle \Psi | G | \Psi \rangle \tag{9}$$

### 2.4. Langevin equation for $D_2$ molecule

Electron localization (weight distribution) of $D_2$ molecule is roughly understood by the normalization equation of wave function,

$$(4\pi)^4 \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \Psi_{2D}^2(r_{A1}, r_{A2}, r_{B1}, r_{B2}) r_{A1}^2 r_{A2}^2 r_{B1}^2 r_{B2}^2 dr_{A1} dr_{A2} dr_{B1} dr_{B2} = 1 \tag{10}$$

$$\int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \rho(r_{A1}, r_{A2}, r_{B1}, r_{B2}) dr_{A1} dr_{A2} dr_{B1} dr_{B2} = 1 \tag{11}$$

$$\rho(r_{A1},r_{A2},r_{B1},r_{B2})=\frac{(4\pi)^4}{2+2\Delta}\begin{Bmatrix}\left[r_{A1}^2\Psi_{100}^2(r_{A1})r_{B2}^2\Psi_{100}^2(r_{B2})\right]r_{A2}^2r_{B1}^2\\+2\left[r_{A1}\Psi_{100}(r_{A1})r_{A2}\Psi_{100}(r_{A2})r_{B1}\Psi_{100}(r_{B1})r_{B2}\Psi_{100}(r_{B2})\right]r_{A1}r_{A2}r_{B1}r_{B2}\\+\left[r_{A2}^2\Psi_{100}^2(r_{A2})r_{B1}^2\Psi_{100}^2(r_{B1})\right]r_{A1}^2r_{B2}^2\end{Bmatrix} \quad (12)$$

Since $|r\Psi|^2$ is the element of particle density localization function, localized peaks appear at $r_{A1}=r_{A2}=r_{B1}=r_{B2}=a=R_B$ (52.9 pm); namely the drawn sphere with Bohr radius is a good measure of electron localization.

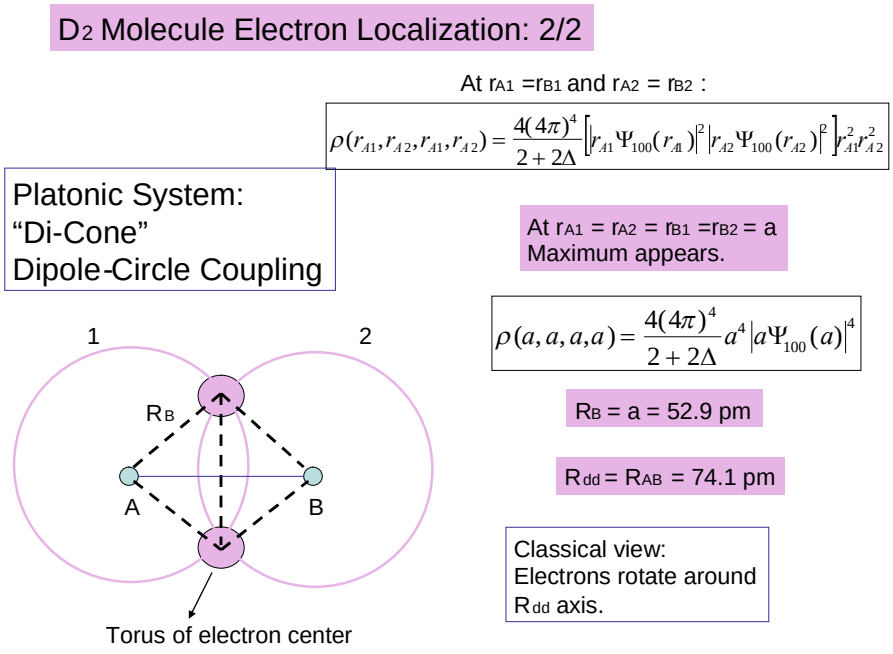



Figure 2. Localization of electron wave and semi-classical image of $D_2$ molecule

In the view of Platonic symmetry, $D_2$ molecule is an orthogonal coupling of d-d line (dipole) and circle (torus of electron center) to form a dicone. Freedom of electron motion is constrained by the existence of counterpart deuteron and electron to form the torus of electron center, but averaged centrifugal force exists as the rotation of electrons around $R_{dd}$ axis. See **Fig. 2**.

The Langevin equation for $D_2$ molecule becomes as,

$$m_d\frac{d^2R_{dd}}{dt^2}=-(4\sqrt{2}-2)\frac{e^2}{R_{dd}^2}+\frac{2m_ev_e^2}{(R_{ee}/2)}-\frac{\partial V_{s2}(R_{dd};1,1)}{\partial R_{dd}}+f(t) \quad (13)$$

Here the Coulomb force term under Platonic symmetry is derived by derivative (minus sign) of Coulomb energy,

$$E_C=-4\frac{e^2}{R_{de}}+\frac{e^2}{R_{dd}}+\frac{e^2}{R_{ee}} \quad (14)$$

$$E_C \approx -4\sqrt{2}\frac{e^2}{R_{dd}} + 2\frac{e^2}{R_{dd}} \tag{15}$$

By taking QM-ensemble average with weight of squared electron wave function,

$$m_d \frac{d^2\langle R_{dd}\rangle}{dt^2} = -\left\langle \frac{5.26}{R_{dd}^2}\right\rangle + 4\left\langle \frac{m_e v_e^2}{R_{ee}}\right\rangle - \frac{\partial V_{s2}(R_{dd};1,1)}{\partial\langle R_{dd}\rangle} + \langle f(t)\rangle \tag{16}$$

The first and second term of Eq. (16) right side cancels each other[4], and we obtain,

$$m_d \frac{d^2\langle R_{dd}\rangle}{dt^2} = -\frac{\partial V_{s2}(R_{dd};1,1)}{\partial\langle R_{dd}\rangle} + \langle f(t)\rangle \tag{17}$$

By taking ensemble average with the Gaussian wave function of d-d pair, the second term of Eq. (17) <f(t)> becomes zero, since we have no distortion in d-d dipole line by QM fluctuation to deviate from the Platonic symmetry. Thus, Eq. (17) becomes well known Newtonian mechanical equation, with constraint by molecular trapping potential $V_{s2}$ ($R_{dd}$;1,1). Mathematical formulas for trapping (shielded) potentials of $D_2$ and $D_2^+$ systems are given in our previous papers[2, 5]. Figure of plotted data for two potentials are shown in **Fig. 3**.

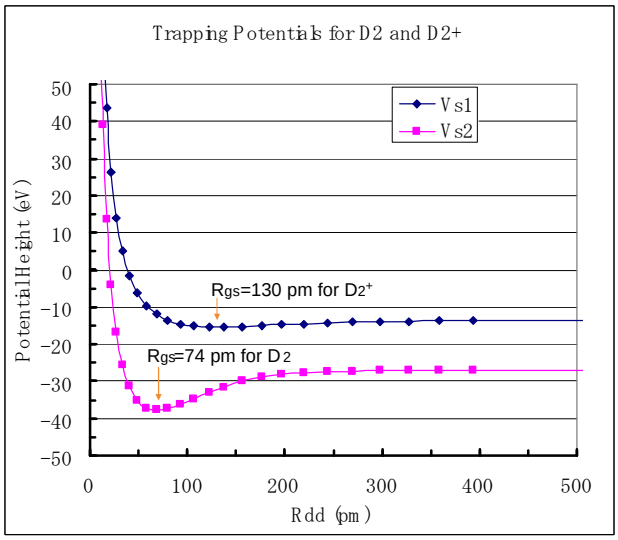



Figure 3. Trapping potential of d-d pair for $D_2$ molecule and $D_2^+$ ion

As understood by potential shape for $D_2$ molecule given in Fig. 3, the Langevin equation for expectation value <$R_{dd}$> gives always convergence to $R_{gs}$ =74 pm after time-dependent motion starting from arbitrary position $R_{dd}$ (t=0). If $R_{dd} > R_{gs}$, we have acceleration force. If $R_{dd} < R_{gs}$, we have deceleration force.

### 2.5. Langevin Equation of $D_2^+$ Ion

In analogy to the $D_2$ molecule, Langevin equation for $D_2^+$ ion (stable in vacuum) is given by,

$$m_d \frac{d^2 R_{dd}}{dt^2} = -2\frac{e^2}{R_{de}^2} + \frac{e^2}{R_{dd}^2} + \frac{m_e v_e^2}{R_e} - \frac{\partial V_s(R_{dd};1,1)}{\partial R_{dd}} + f(t) \tag{18}$$

By taking QM-ensemble average,

$$-e^2\left\langle \frac{2}{R_{de}^2} - \frac{1}{R_{dd}^2} \right\rangle + \left\langle \frac{m_e v_e^2}{R_e} \right\rangle = 0 \tag{19}$$

$$m_d \frac{d^2\langle R_{dd}\rangle}{dt^2} = -\frac{\partial V_s(R_{dd};1,1)}{\partial \langle R_{dd}\rangle} \tag{20}$$

Potential curve is shown in Fig. 3. In the view of Platonic symmetry, $D_2^+$ ion molecule is an elongated dicone with $R_{dd}$=130 pm, with rotating triangle of "d-e-d" type face around the $R_{dd}$ axis. Dynamic motion of deuteron by Eq. (20) gives convergence to $R_{dd}=R_{gs}$=130 pm.

In the following for complex D-cluster systems, Eq. (17) and Eq. (20) with those potentials will provide intrinsic components of friction (constraint) by QM electron waves with D-cluster condensation.

### 2.6. Langevin Equations of Expectation Values for Complex D-Clusters

In complex D-cluster systems under Platonic symmetry, averaged rotation motion over whole system is prohibited by constraints of many particle arrangements. This *form*[3] of self-organization makes simpler treatment to derive one-dimensional Langevin equation possible. The term *form* is meta-physical concept.

The QM-ensemble average on electron wave function can be subdivided as multiple constraint function of "d-e-d" type or "d-e-d-e" type potential derivative as,

$$\langle Constra\,\mathrm{int}\rangle_{electron-wave} = -N_f \frac{\partial V_{si}(R_{dd};1,1)}{\partial R_{dd}} \tag{21}$$

Here $N_f$ is the number of faces of Platonic polyhedron for a D-cluster, and i=1 for the "d-e-d" type ($D_2^+$ type) face and i=2 for the "d-e-d-e" type ($D_2$ type) face.

The Langevin equation for a D-cluster under Platonic symmetry with $N_e$ number of d-d edges and $N_f$ number of faces is written for $N_e$ >2,

$$N_e m_d \frac{d^2 R}{dt^2} = -\frac{k}{R^2} - N_f \frac{\partial V_s}{\partial R} + f(t) \tag{22}$$

Here k is constant (11.8 for 4D/TSC).

The QM-ensemble average on d-d wave function (assuming Gaussian form) is given by

$$N_e m_d \left\langle \Psi(R,R') \left| \frac{d^2 R}{dt^2} \right| \Psi(R,R') \right\rangle = -\left\langle \Psi(R,R') \left| \frac{k}{R^2} \right| \Psi(R,R') \right\rangle$$
$$- N_f \left\langle \Psi(R,R') \left| \frac{\partial V_s}{\partial R} \right| \Psi(R,R') \right\rangle + \left\langle \Psi(R,R') | f(t) | \Psi(R,R') \right\rangle \quad (23)$$

with a Gaussian wave function for d-d pair of D-cluster,

$$\Psi(R,R') = \frac{1}{\sqrt{2\pi\sigma^2}} \exp(-(R'-R)^2/(2\sigma^2)) \quad (24)$$

We drive a Langevin equation for expectation value $<R_{dd}> = <R>$ as,

$$N_e m_d \frac{d^2 < R >}{dt^2} = -\frac{k}{R^2} - N_f \frac{\partial V_s}{\partial R} + < f(t) > \quad (25)$$

For complex D-cluster, <f(t)> value does not always zero because of deviation of D-cluster system from ideal Platonic symmetry, due to the quantum mechanical fluctuation of d-positions which may distort the Platonic symmetry. The perturbed force component by this QM distortion is approximately given by the next formula, which is the change of system Coulomb energy derivative, as,

$$-\frac{\partial \Delta E_C}{\partial R} \approx -k \frac{1}{2} \left( \frac{1}{(R+\Delta R)^2} + \frac{1}{(R-\Delta R)^2} \right) + k \frac{1}{R^2}$$
$$\approx -\frac{k}{2R^2} \left( 1 - \frac{2\Delta R}{R} - \left(\frac{\Delta R}{R}\right)^2 + 1 + \frac{2\Delta R}{R} - \left(\frac{\Delta R}{R}\right)^2 \right) + \frac{k}{R^2} \quad (26)$$
$$= \frac{k}{R^2} \left( \frac{\Delta R}{R} \right)^2$$

By using a Gaussian squared wave function for d-d pair fluctuation, we write,

$$(\Delta R)^2 = (\sigma R)^2 \quad (27)$$

The change of Coulomb force by distortion is given by

$$<\text{Change of Coulombic Force}> = \frac{k\sigma^2}{(R_{dd})^2} \quad (28)$$

### 2.7. Langevin Equation of $D_3^+$ Ion Molecule

It is well known that tri-atomic hydrogen molecular ion $D_3^+$ (or $H_3^+$) is generated in ion source and glow discharged plasma and very stable in vacuum. However, quantum molecular physics for the system is of difficult problem to solve and studies are being continued in astrophysics needs.

The analogy of the present methodology for D (or H)-cluster under orthogonal coupling of Platonic symmetries for electrons and deuterons (or protons) can provide rather simple way of modeling its dynamics. Applying Eq. (25), we obtain,

$$3m_d \frac{d^2\langle R_{dd}\rangle}{dt^2} = -\frac{6.13}{\langle R_{dd}\rangle^2} - 6\frac{\partial V_s(R_{dd};1,1)}{\partial\langle R_{dd}\rangle} + \langle f(t)\rangle \tag{29}$$

Here the force is given with unit of [keV/pm]. Image of $D_3^+$ ion is given in **Fig. 4**.

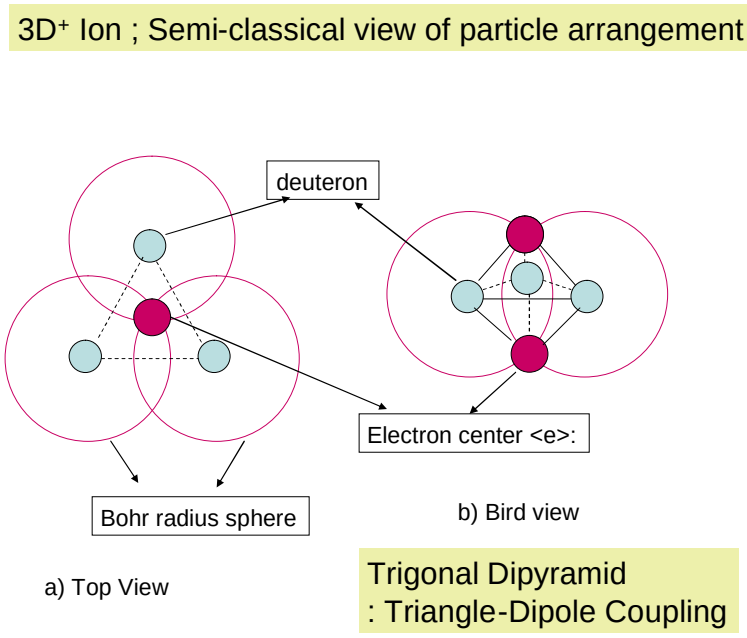



Figure 4. Tri-atomic hydrogen (deuterium) molecular ion and Platonic arrangement

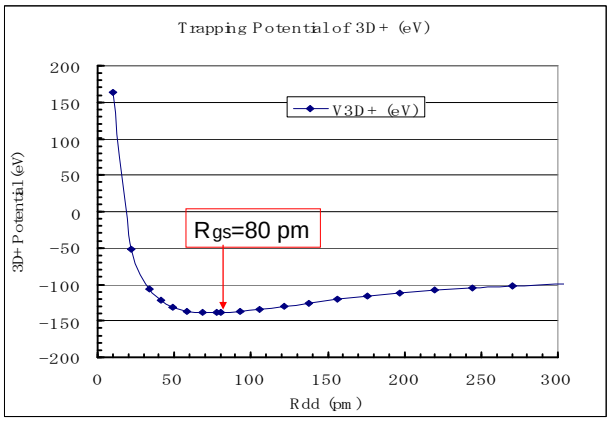
The system Coulomb energy and its derivative can be calculated by simple geometry exercise for the Platonic symmetry system of trigonal di-pyramid which is the orthogonal coupling of the 3d regular triangle and the <e> - <e> line (dipole). Here two electron centers (or electron balls) appear in the system, and system-averaged rotation of electrons is prohibited (no averaged centrifugal force).

By distortion of ideal Platonic symmetry with QM fluctuations of deuteron positions, we have positive <f(t)> bias. As we have 3 d-d edges in the system, 3 times of Eq. (28) becomes the bias

(about 30% of main Coulomb acceleration force). Therefore the total potential of the system becomes in expected value equation, as,

$$V_{3D+main}(R_{dd}) = -\frac{6.13}{R_{dd}} + 6V_s(R_{dd};1,1) + (\text{<f(t)> component}) \tag{30}$$

The calculated curve of this potential is shown in **Fig. 5**.




$$3m_d \frac{d^2\langle R_{dd}\rangle}{dt^2} = -\frac{6.13}{\langle R_{dd}\rangle^2} - 6\frac{\partial V_s(R_{dd};1,1)}{\partial\langle R_{dd}\rangle} + \langle f(t)\rangle$$

$$\langle f(t)\rangle = \left\langle -\frac{\partial \Delta E_C}{\partial R'} \mathrm{X}^2(R_{dd};\sigma,t) \right\rangle$$

:Distortion of Coulomb force from 3D
Regular triangle arrangement (about 30%)

Figure 5. Trapping potential of $D_3^+$ ion molecule with Langevin equation for expectation value of d-d distance of 3d regular triangle

The tri-atomic hydrogen ion is thus stable and has its ground state at $R_{gs}$= 80 pm. As a reference, Helm et al (Freiburg University, 2003; google triatomic hydrogen ion and Helm) gave about $R_{gs}$ = 85 pm[8] which agrees considerably well with our result taking into account that appropriate sigma-value of wave function[2] is about 30 % of $R_{dd}$.

We can conclude that our approach with one-dimensional Langevin equations for D-cluster systems look successful.

## 3. Langevin Equation for 4D/TSC and Numerical Solution

### 3.1. Double Platonic Symmetry

In **Fig. 6**, we show feature of electron cloud for 4D/TSC (t=0), compared with those of D-atom and $D_2$ molecule.

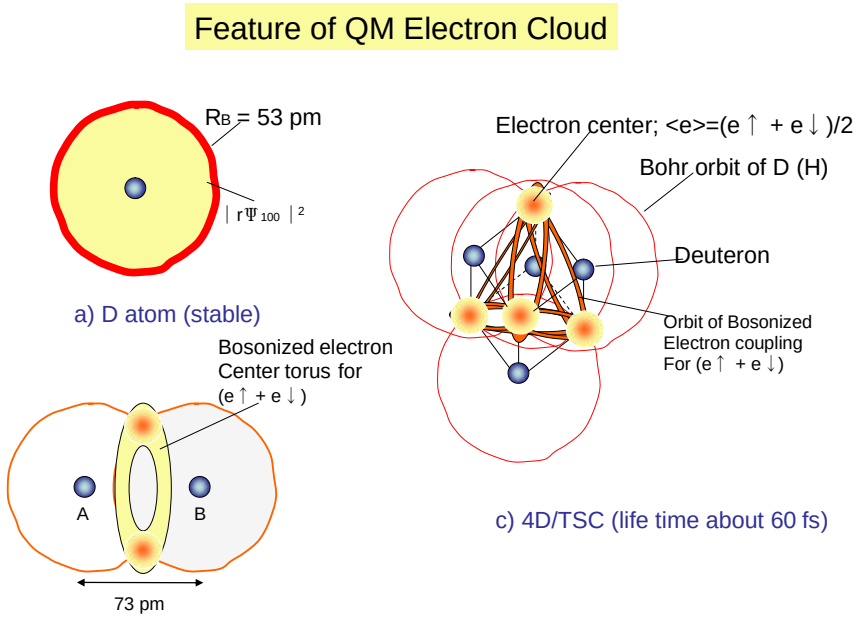
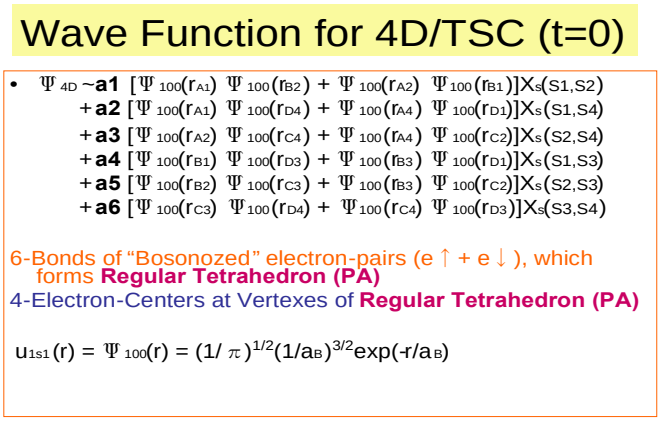



Figure 6. Feature of QM electron clouds for 4D/TSC (t=0), compared with those of D-atom and $D_2$ molecule

The *form*[3)] of 4D/TSC (t=0) wave function is given[1,3,4)] as,




Top equation (31)

This TSC system has double symmetry of regular tetrahedrons for deuterons and electron-centers, namely the *double Platonic symmetry* which is the most ideal system in 3-dimensional condensation squeezing into the central focal point (Center-of-Mass; CMS).

### 3.2. Langevin Equation for 4D/TSC

The system Coulomb energy and its derivative are given in our previous work[1,2)]. We write here resulting final Langevin equation for Monte-Carlo calculation.

$$6m_d \frac{d^2 R_{dd}(t)}{dt^2} = -\frac{11.85}{[R_{dd}(t)]^2} - 6\frac{\partial V_{s2}(R_{dd}(t);1,1)}{\partial R_{dd}(t)} + \langle f(t)\rangle + f'(t) \tag{32}$$

with

$$f'(t) = f(t) - \langle f(t)\rangle \tag{33}$$

$$f(t) = [-\frac{\partial \Delta E_c(R_{dd})}{\partial R_{dd}}] \mathrm{mod}[X^2(R'_{dd}; R_{dd}(t))] \tag{34}$$

$$\mathrm{X}^2(R'_{dd}; R_{dd}(t)) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp[-(R'_{dd} - R_{dd}(t))^2/(2\sigma^2)] \tag{35}$$

For QM-ensemble averaged equation, we obtained [2],

$$6m_d \frac{d^2\langle R_{dd}\rangle}{dt^2} = -\frac{11.85}{\langle R_{dd}\rangle^2} - 6\frac{\partial V_s(\langle R_{dd}\rangle; m, Z)}{\partial\langle R_{dd}\rangle} + 6.6\left\langle \frac{(R'-R_{dd})^2}{R_{dd}{}^4}\right\rangle \tag{36}$$

Time-dependent potential for this equation was given[2] as,

$$V_{tsc}(R': R_{dd}(t)) = -\frac{11.85}{R_{dd}(t)} + 6V_s(R_{dd}(t); m, Z) + 2.2\frac{|R'-R_{dd}(t)|^3}{[R_{dd}(t)]^4} \tag{37}$$

Here we fixed m=2 and Z=2 for $V_s$ (dde*(2,2) potential) for numerical calculation.

The third term of right side of Eq. (36) gives about 15% positive bias to main Coulomb force (first term), and was merged[2] in the first term by multiplying factor 0.85 in the numerical calculation by the Verlet method.

In **Fig. 7**, we show the calculated trapping potential of 4D/TSC, compared with that of $6D^{2-}$/OSC (shown later). 4D(or H)/TSC keeps in average the always accelerating force in its condensation motion, hence it can condense ultimately until when TSC-min state (about 10-20 fm radius) comes, as illustrated in **Fig. 8**. On the contrary, $6D^{2-}$/OSC converges to $R_{dd}$=40 pm on the way of condensation (we derive equation later). *Within the presently studied 5 kinds of D-clusters, only 4D/TSC can condense ultimately to very small charge neutral entity.*

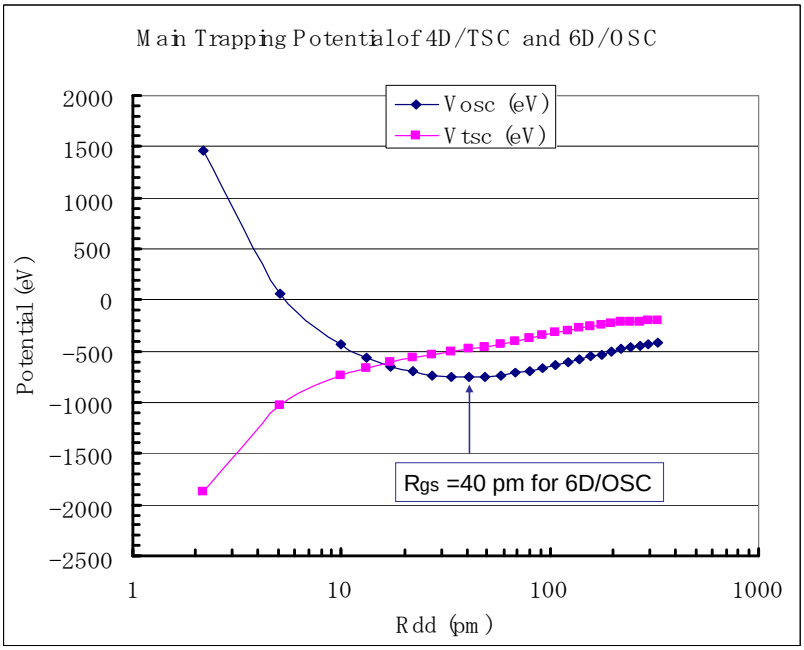



Figure 7. Trapping potential of 4D(or H)/TSC, always attractive, compared with $6D^{2-}$/OSC potential which has converging point (40 pm) at its ground state

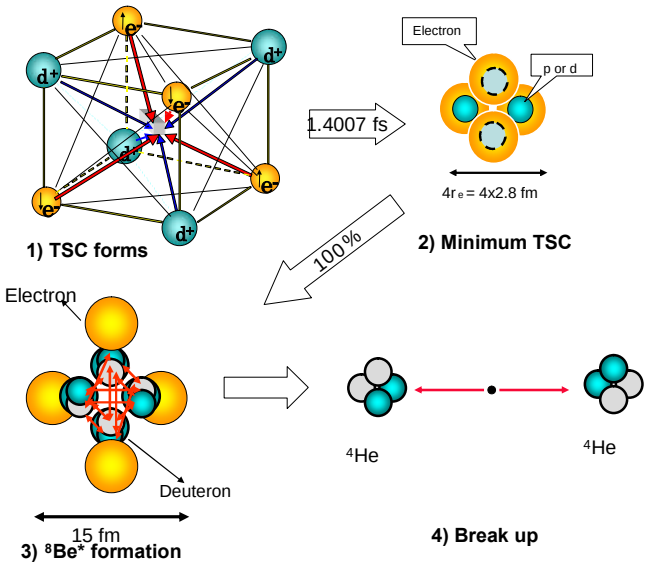



Figure 8. Condensation of 4D/TSC and 4D-fusion to two $^4$He-particles break-up

Numerical solution of Eq. (36) was obtained by a computer code based on the Verlet time-step method[2], a standard result is shown in **Fig. 9**.

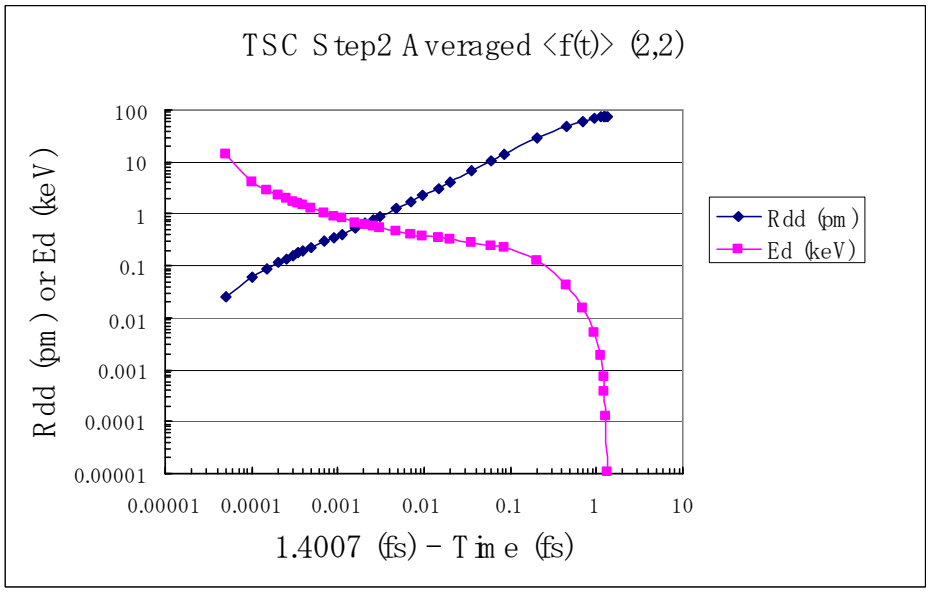



Figure 9. Numerical results of 4D/TSC condensation motion; time-variation of $<R_{dd}>$ and mean deuteron kinetic energy $<E_d>$

The condensation time of 4D/TSC is very fast as 1.4007 fs. As we show in the next section, 4D fusion reaction takes place with almost 100% probability in the final stage of condensation within the time interval of about $2x10^{-20}$ s. For other details of time-dependent behavior of TSC dynamics with different conditions, see our previous paper[2].

### 3.3. Estimation of Fusion Rates

The Gaussian wave function was quite useful for making QM-ensemble averaging of Langevin equation, but unfortunately the accuracy in its tails (for very small $R_{dd}$ values) is not high enough to apply for the estimation of barrier factor of d-d pair and 4d cluster, as we illustrate the relation between trapping (shielded) Coulomb potential, wave function and the very short (within $r_0$ = about 5 fm) range of nuclear strong interaction, in **Fig. 10**.

To obtain usable accuracy in barrier factor, we introduced the Heavy Mass Electronic Quasi-Particle Expansion Method (HMEQPET) to provide equivalent time-dependent potentials for the squeezing 4D/TSC system. Detail description is given in our previous paper[2].

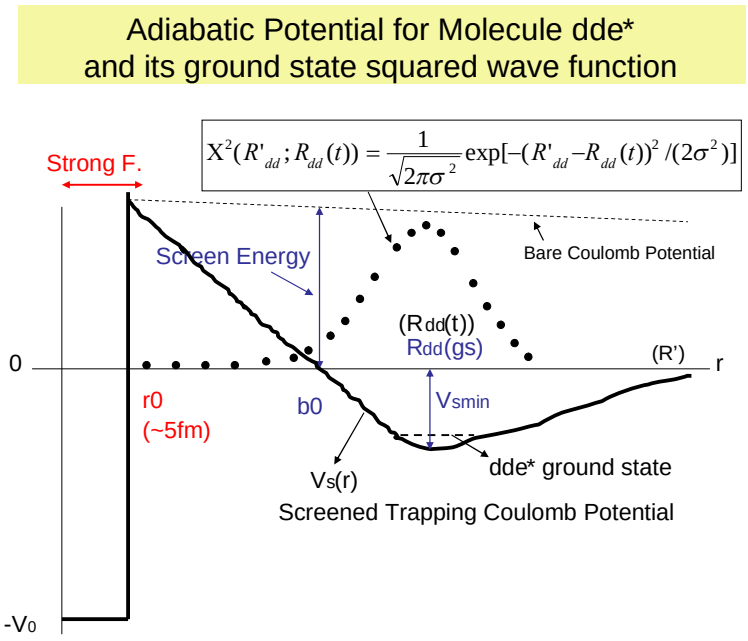



Figure 10. Relation between d-d trapping (shielded) Coulomb potential, wave function and the very short (within about 5 fm) range of nuclear strong interaction. By calculating Gamow integral from $R_{gs}$ to $r_0$, we obtain barrier factor for fusion rate estimation.

Time-dependent or equivalently $R_{dd}$-dependent barrier factors were calculated as we copy from our previous paper[2] as given in **Table-1**. TSC starts at $R_{dd}$=74 pm and condenses very rapidly to reach at the final TSC-min state with $R_{dd}$ = 0.0206 pm (20.6 fm) in this case. On the way of condensation, TSC passes the equivalent state with that of dd-muon molecule for which we have reference data[6] to show good agreement with our calculation. Barrier factors for 2d and 4d fusion were calculated using the WKB approximation with Gamow integral[2].

Fusion rates for steady molecules were then calculated based on the Fermi's first golden rule[2],

$$\lambda_{nd} = \frac{2}{\hbar}\langle W\rangle P_{nd}(r_0) = 3.04\times10^{21} P_{nd}(r_0)\langle W\rangle \tag{38}$$

Here $P_{nd}$ is barrier factor for nD-cluster and <W> is the averaged value of imaginary part of nuclear optical potential[7]. The extrapolation of <W> value to 4d fusion was made[2] by using the scaling law $\langle W\rangle \propto (PEF)^5$ with PEF-value which is given in unit of derivative of one pion exchange potential (OPEP) (simple case of Hamada-Johnston potential[10] for pion exchange model) given by

$$\langle OnePEF\rangle = -\frac{\partial\langle V_{OPEP}(x)\rangle_{\tau,\sigma}}{\partial r} = -\frac{1}{1.43}\frac{\partial\langle V_{OPEP}(x)\rangle_{\tau,\sigma}}{\partial x} \tag{39}$$

$$V_{OPEP}(x) = v_0 \cdot (\vec{\tau}_1 \cdot \vec{\tau}_2)\left\{\vec{\sigma}_1 \cdot \vec{\sigma}_2 + (1 + \frac{3}{x} + \frac{3}{x^2})S_{12}\right\}\frac{\exp(-x)}{x} \quad (40)$$

Here $x = \frac{m_\pi c}{\hbar} r = \frac{r}{1.43}[fm]$

and $S_{12}$ is the tensor operator $S_{12} = 3\frac{(\vec{\sigma}_1 \cdot \vec{r})(\vec{\sigma}_2 \cdot \vec{r})}{r^2} - \vec{\sigma}_1 \cdot \vec{\sigma}_2$

and $v_0 = \frac{1}{3}\frac{f^2 m_\pi c^2}{\hbar c} = 3.65[MeV]$.

And $\vec{\tau}$ is the isospin operator for n-p charged pion exchange and $\vec{\sigma}$ is the spin operator for nucleon state. The table of <W> values is given in our previous paper[2].

Time-integrated fusion yield per TSC generation was given[2] by the following formulas,

$$\eta_{4d} = 1 - \exp(-\int_0^{t_c} \lambda_{4d}(t)dt) \quad (41)$$

$$\lambda_{4d}(t) = 3.04 \times 10^{21} \langle W \rangle P_{4d}(r_0; R_{dd}(t)) = 1.88 \times 10^{23} P_{4d}(r_0; R_{dd}(t)) \quad (42)$$

$$\int_0^{t_c} \lambda_{4d}(t)dt = 1.88 \times 10^{23} \int_0^{t_c} P_{4d}(r_0; R_{dd}(t))dt \quad (43)$$

$$\int_0^{t_c} P_{4d}(r_0 : R_{dd}(t)dt = 2.31 \times 10^{-22} \quad (44)$$

$$\eta_{4d} \cong 1.0 \quad (45)$$

Macroscopic fusion rate is given by

$$Y_{4d} = Q_{tsc}\eta_{4d} \quad (46)$$

We have obtained that 4D fusion may take place with almost 100 % yield per a TSC generation, so that macroscopic 4d fusion yield is given by simply with TSC generation rate $Q_{tsc}$ in the experimental conditions of CMNS. However, when we consider that one deuteron has spin-parity 1+ and combination of 4d has total spin state 4, 3, 2, 1 and 0, the 4d fusion with out-going channel to two $^4$He (0+:gs) particles is forbidden, by spin-parity conservation (for S-wave in/out channels), except for the 0+ spin-parity state (T=0) of 4d combination, to be explained detail analysis including P-wave and D-wave states with isospin elsewhere.

Table-1: barrier factors under 4D/TSC condensation

**Barrier Factors of 4D/TSC under condensation, Calculated by HMEQPET Code**

| $R_{dd}=R_{gs}$ (pm) | $P_{2d}$ ; 2D Barrier Factor | $P_{4d}$; 4D Barrier Factor |
|---|---|---|
| 0.0206 (TSC-min) | 4.44E-2 | 1.98E-3 |
| 0.0412 | 1.06E-2 | 1.12E-4 |
| 0.103 | 1.43E-3 | 2.05E-6 |
| 0.206 | 3.35E-5 | 1.12E-9 |
| 0.412 | 9.40E-7 | 2.16E-13 |
| 0.805 ( $\mu$ dd) | 1.00E-9 | 1.00E-18 |
| 1.03 | 9.69E-11 | 9.40E-21 |
| 2.06 | 6.89E-15 | 4.75E-29 |
| 4.12 | 9.38E-21 | 8.79E-41 |
| 10.3 | 2.16E-32 | 4.67E-64 |
| 21.8 (dde*(2,2) | 1.30E-46 | 1.69E-92 |
| 74.1 (D2 molecule) | 1.00E-85 | 1.00E-170 |

Table-2: Fusion rates by Fermi's golden rule for steady molecules

| Molecule | $R_{dd}=R_{gs}$ (pm) | $P_{nd}$ ; B-Factor | <W> (MeV) | $\lambda_{2d}$ (f/s) | $\lambda_{4d}$ (f/s) |
|---|---|---|---|---|---|
| $D_2$ | 74.1 | 1.0E-85 | 0.008 | 2.4E-66 | |
| dde*(2,2) | 21.8 | 1.3E-46 | 0.008 | 3.16E-27 | |
| $\mu$ dd | 0.805 | 1.0E-9 | 0.008 | 2.4E+10 | |
| 4D/TSC-min | 0.021 | 1.98E-3 | 62 | | 3.7E+20 |

The ultimate condensation is possible only when the double Platonic symmetry of 4D/TSC is kept in its dynamic motion. The sufficient increase (super screening) of barrier factor is also only possible as far as the Platonic symmetric 4D/TSC system is kept. Therefore, there should be always 4 deuterons in barrier penetration and fusion process, so that 4d simultaneous fusion should take place predominantly. The portion of 2D (usual) fusion rate is considered to be negligible[2)].

Major nuclear products of 4D fusion are two 23.8 MeV α-particles[5,7)]. 4H/TSC should condense in the same way until when TSC-min state with classical electron radius (2.8 fm) comes, but no strong interaction exists among protons and will make 1p to 4p capture transmutations with host metal nuclei when 4H/TSC has sufficient drift (CMS) momentum.

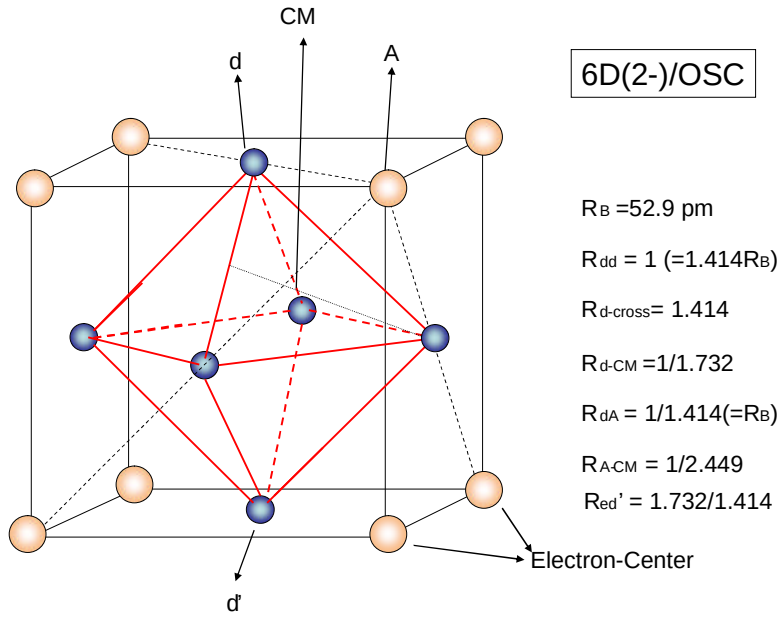



Figure 11. Illustration of $6D^{2-}$/OSC system

## 4. Langevin Equation for $6D^{2-}$/OSC

To fulfill the orthogonally coupled Platonic symmetry for 6D-cluster, 8 electron centers should appear on the center lines of 8 regular triangle faces of 6d octahedron; see **Fig. 11**. Therefore, the Platonic OSC should be with 2- negative ion state.

The Langevin equation for $6D^{2-}$/OSC becomes as,

$$12m_d \frac{d^2R_{dd}(t)}{dt^2} = -\frac{29.3}{[R_{dd}(t)]^2} - 24\frac{\partial V_s(R_{dd}(t);1,1)}{\partial R_{dd}(t)} + \langle f(t)\rangle + f'(t) \qquad (47)$$

The effective trapping potential of this system was already given in Fig. 7, which tells us that $6D^{2-}$/OSC does not make ultimate condensation. However, in transient condensation process, we may have small probability that d-d distance would approach in shorter d-d distances than 40 pm of its ground state and 6D fusion rate may be somehow enhanced. We need numerical study for this.

We need a different study on if there exists a condensing system of neutral 6D-cluster (face-centered dodecahedron[11]) by coupling of two octahedrons (one of 6 deuterons and the other of 6 electron-centers).

## 5. Conclusions

1) Platonic Symmetric Arrangement realizes Energy-Minimum State of Many-Body System.

2) Platonic symmetry appears in D-atom, $D_2$, $D_2^+$,$D_3^+$ molecule, and 4D/TSC.

3) Platonic symmetry appears in CMNS of 4D/TSC for both of the Coulombic interaction and the strong interaction.

4) Dynamic Platonic symmetry is of key for super-screening of Coulomb repulsion and 4D Cluster Fusion.

5) We have obtained good solutions of molecular dynamics with Langevin Equations, for Platonic symmetric systems as, D, $D_2$, $D_3^+$ and 4D(or H)/TSC.

6) It was shown that about 100% 4D-fusion per TSC generation is possible, by the present work, in the condensed matter nuclear effects.

7) Only 4D (or H)/TSC can condense ultimately to a very small charge neutral entity with 10-20 fm radius size, as far as 5 kinds of D-clusters studied in this work.

8) Bosonized e( 1 /2)+e(-1/2) coupling for the "d-e-d-e" system makes $D_2$ type faces of 4D(or H)/TSC to help its ultimate condensation.

9) $6D^{2-}$/OSC converges its condensation at about $R_{dd}$=40 pm, but closer d-d distance in transient may appear with small probability.

10) Single <e- >-center states for the "d-e-d" ($D_2^+$) type faces of $D_3^+$ ion molecule and $6D^{2-}$/OSC enhance constraint (friction) for their condensation.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