How the process changes from $d + d \rightarrow t + p$ (and $^3He + n$) to $d + d \rightarrow {}^4He$

Tetsuo SAWADA (RIMM)

In the low energy limit of the d + d reaction in vacuum , there appear three types of final states, they are :

1. $d + d \rightarrow$ t + p br. ratio $\approx$ 0.5
   (25%) (75%)

2. $d + d \rightarrow$ ^{3}He + n br. ratio $\approx$ 0.5
   (25%) (75%)

3. $d + d \rightarrow$ ^{4}He + γ br. ratio ⊙ 10^{-6}
   (0.32 %) (99.68%)

The energy conservation

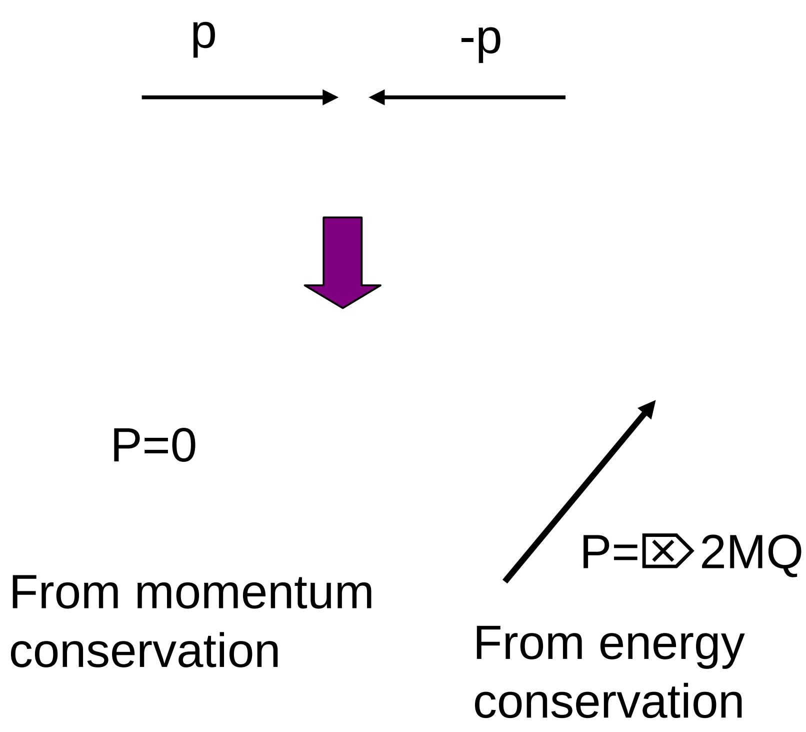
$$Q = p^2/2m_1 + p^2/2m_2 \quad \text{or} \quad Q = pc + p^2/2m$$

is used, where Q is the energy produced in the reaction.

Implication of $d + d \to {}^4He$ process (I)

In general, the process "two body $\to$ one body" is forbidden, because it cannot satisfy the conservation laws of momentum and energy simultaneously.

- To see this fact more clearly, let us compute the momentum p of the final state particle. If we see the reaction in the center of mass system, the momentum conservation requires p=0. On the other hand, the energy conservation requires $p^2 = 2MQ$, where Q is the energy produced in the reaction . Therefore these results are not compatible if Q is positive.



Implication of d + d $\rightarrow$ ^{4}He process (II)

- When we start to learn the quantum field theory, the first thing we learn is the Noether's theorem. It says that if the system has translational invariance in time t, the energy conservation arises. On the other hand, the momentum conservation comes from the translational invariance in space **r**.
- In the problem of the potential scattering, the scattering amplitude $f(\vec{q})$ is given by the Fourier transformation :

$$f(\vec{q}) = -(2m/4\pi\hbar^2)\int V(r')\exp[i\vec{q}\cdot\vec{r}']d^3r'$$

- The probability of such a process is $|f|^2$, in which the momentum transfer $\vec{q} = \vec{p}_f - \vec{p}_i$ is absorbed by the potential.

Implication of d + d $\rightarrow$ ^{4}He process (III)

- For our case of ^{4}He production, since Q=23.9MeV. and Mc^2=3727MeV. the momentum transfer q becomes q=422MeV. /c, so the size (locality) of the external potential V(r) is severely restricted. From the uncertainty relation the spread of V(r) is

$$\Delta r \approx \hbar / q = 0.47 \, fm .$$

- Therefore the electron cloud, whose size is order of Å, cannot do the job to absorb such a large momentum transfer.

On the ability to absorb the momentum transfer q

Let us compute the Fourier transformation of two external potentials:

- Coulomb potential produced by the Gaussian charge distribution

  $\rho(r) = Q \exp[-r^2/r_0^2]/(r_0\sqrt{\pi})^3$ is

  $$V_g(r) = (Q/r)(2/\sqrt{\pi})(\,\mathrm{erf}\,(r/r_0) - (r/r_0)\exp[-r^2/r_0^2])$$

  and whose Fourier transformation becomes:

  $$\tilde{V}_g(q) = (4\pi Q/q^2)\exp[-q^2 r_0^2/4](1 - q^2 r_0^2/2)$$

  , therefore the electron cloud of pm.-size cannot absorb q.

- The potential of the dipole moment in the magnetic Coulomb field is

  $$V_{dip}(r) = \kappa(D/4m)\left\{1 - (1 + ar + a^2 r^2/2)\exp[-ar]\right\}/r^2$$

  , where a=6.04, and whose Fourier transformation is:

$$\tilde{V}_{dip}(q) = \frac{C'}{f}\left\{\frac{\pi}{2} - \mathrm{atn}\; f - \frac{f + \sin(2\,\mathrm{atn}\; f)/2}{(1 + f^2)}\right\} \quad with \quad f = \frac{q}{a}$$

where $C' = 4\pi\kappa(D/4m)/a$. At f=0.501 the value of the Fourier transformation is $\tilde{V}_{dip}/C' = 0.77$, therefore the nucleon can absorb the momentum transfer q and make it possible to produce ^{4}He.

How the magnetic monopole is selected as the candidate of the catalyst of the cold nuclear fusion

- We learned that the external field to “absorb” the momentum transfer q is necessary to realize the cold nuclear fusion .
- However there are only three types of external fields to which the nuclei can respond. They are (1) the electric field, (2) the magnetic field and (3) the pionic field. The sources of these fields are the catalyst particles.
- In addition to speed up the reaction, the catalyst particle must have the properties that it attracts the fuel particles and expels the product particles. Moreover it is desirable for the attractive force to be long range in order to gather the

  surrounding fuel nuclei..

Selection of the candidate of the catalyst particle

- From these criteria, (1) and (3) are excluded, because:
- (1) It is true that the large negative charge can attract small nuclei such as p, d, t and ^{3}He, however it attracts the product nucleus ^{4}He also.
- (3) the source particle of the pionic field is other nucleus, it gives rise to the nuclear force. However it does not behave oppositely to the fuel nuclei and to the product ^{4}He. Moreover, the nuclear force is short range and it cannot attract and gather the surrounding fuel nuclei.

- (2) The magnetic field, whose source particle is the magnetic monopole, satisfies all the necessary requirements. It attracts the fuel nuclei with magnetic moments such as p, n, d, t and ^{3}He, and it repels the spin-0 charged particle such as ^{4}He . Moreover the attractive force is long range: $V \propto -c/r^2$.

Hamiltonian of the magnetic monopole-nucleon system

- Since the magnetic monopole is the unique candidate of the catalyst of the nuclear fusion reaction, it is worthwhile to study the system of monopole plus a nucleus(A) by solving the Schrödinger equation in the magnetic Coulomb field, or by simulating the equation. The Hamiltonian of the monopole-nucleon system is

$$H_{m-N}=\frac{1}{2m_N}(-i\vec{\nabla}-Ze\vec{A})^2-\kappa_{tot}\frac{{}^*ee}{2m_p}(\vec{\sigma}\cdot\hat{r})\frac{F(r)}{r^2}$$

, where $\vec{A}$ is the vector potential whose rotation is the magnetic Coulomb field. F(r) comes from the form factor of the nucleon, and its form is:

$$F(r)=1-(1+ar+a^2r^2/2)\exp[-ar] \quad with \quad a=6.04\mu_\pi$$

Hamiltonian of the magnetic monopole-nucleus (A) system

- It is straightforward to extend the Hamiltonian to nucleus of A nucleons plus monopole system:

$$H_A = \sum_{i=1}^{A} H_{m-N}^{(i)} + \sum_{i>j} V_{i,j}$$

, where $V_{i,j}$ is the nuclear potential between i-th and j-th nucleons. In the simulation of our quantum system, the necessary computing time increases very rapidly with the nucleon number A. So we consider only the small nuclear system: A◙4.

However such a small system (A◙4) is already interesting enough, because we can see the cold nuclear fusion such as p + t $\rightarrow$ ^{4}He or d + d $\rightarrow$ ^{4}He proceeds in our computer by solving the time dependent Schrödinger equation.

Proposal to extend the nuclear physics to ….(I)

- It is instructive to remember the history of the nuclear physics briefly. It started as the model-making , such as the liquid drop model etc., of the proton-neutron system. It experienced two important changes.

- In 1960’s , the hyperons such as Λ , Σ and Ξ are added as the new ingredients to p and n. Today hyper-nucleus is the important branch of the nuclear physics.

- In 1990’s , people started to construct nucleus from the first principle, namely from the nuclear potential, by solving a few body system. Today we can treat the system of A⭘12 or 13. The most important findings of such an approach is the confirmation of the three-body nuclear force, which was predicted by Fujita-Miyazawa in 1950’s.

- Our proposal is to extend the nuclear physics to include the magnetic monopole *e as the additional ingredient.
- The experiences to make the nucleus from the first principle are helpful in constructing the (p, n,*e) world from the known Hamiltonian.
- The first thing we have to do is to determine the energy eigen-values and their wave functions of the bound states of the *e-nucleus system. The results of the calculation are listed .
- Next thing we must do is to compute the transition between various states. For example, when the nucleon number A=4, there appear states |1>=|d, d, *e>, |2>=|t, p, *e>, |3>=|^{3}He, n, *e> and |4>=|^{4}He, *e>. The state Ψ oscillates among these states. More precisely, if we write

$$\Psi(t) = c_1(t)\,|1> + c_2(t)\,|2> + c_3(t)\,|3> + c_4(t)\,|4>$$

the coefficients $\vec{c}(t)$ change according to the following equation:

Proposal to extend the nuclear physics to ….(3)

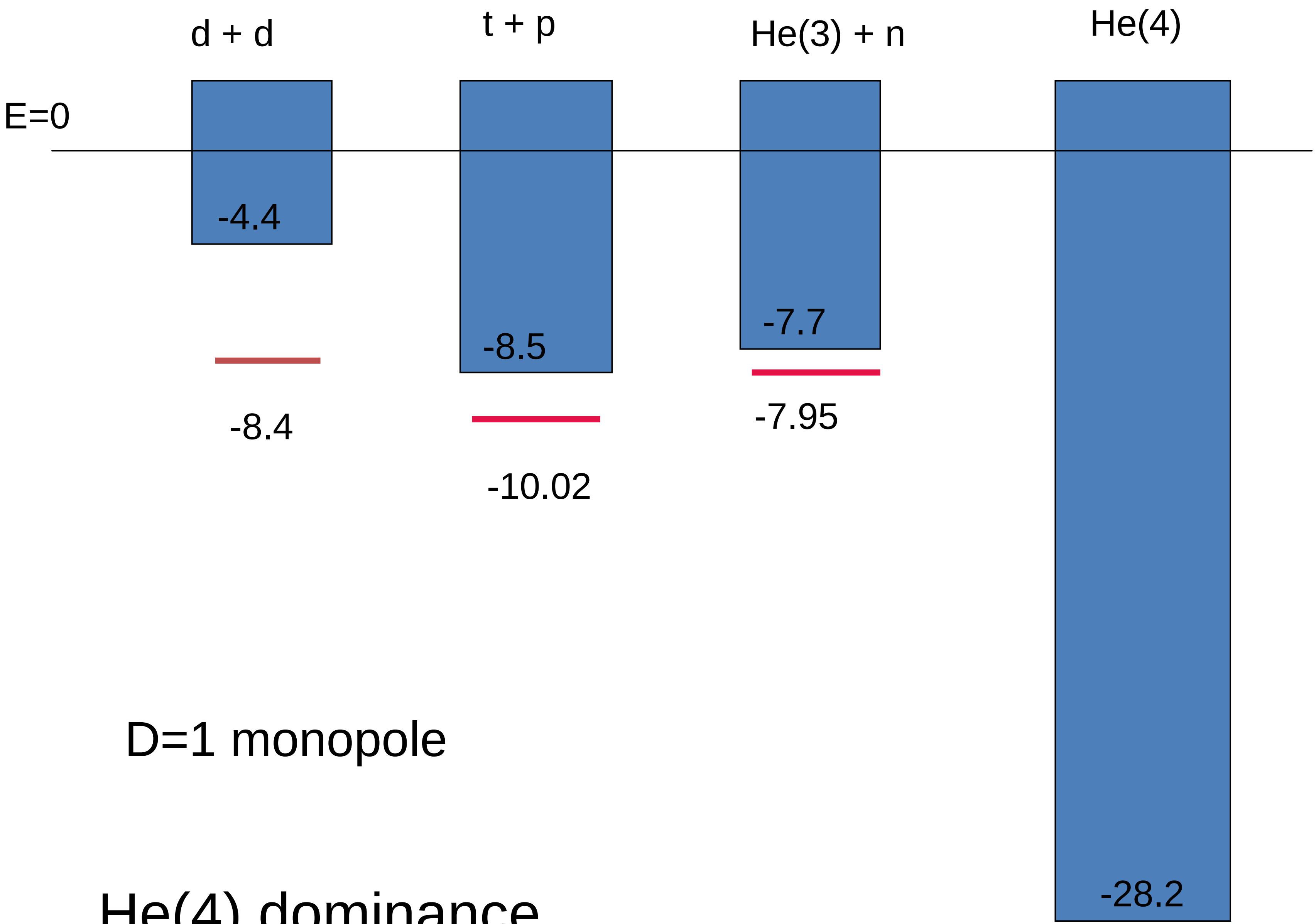
- The equation of transition is:

$$-i\hbar \frac{\partial}{\partial t}\vec{C}(t) = M\,\vec{C}(t)$$

, where M is the transition matrix , whose elements are <i|V|j > , in which V is the sum of the nuclear potentials.

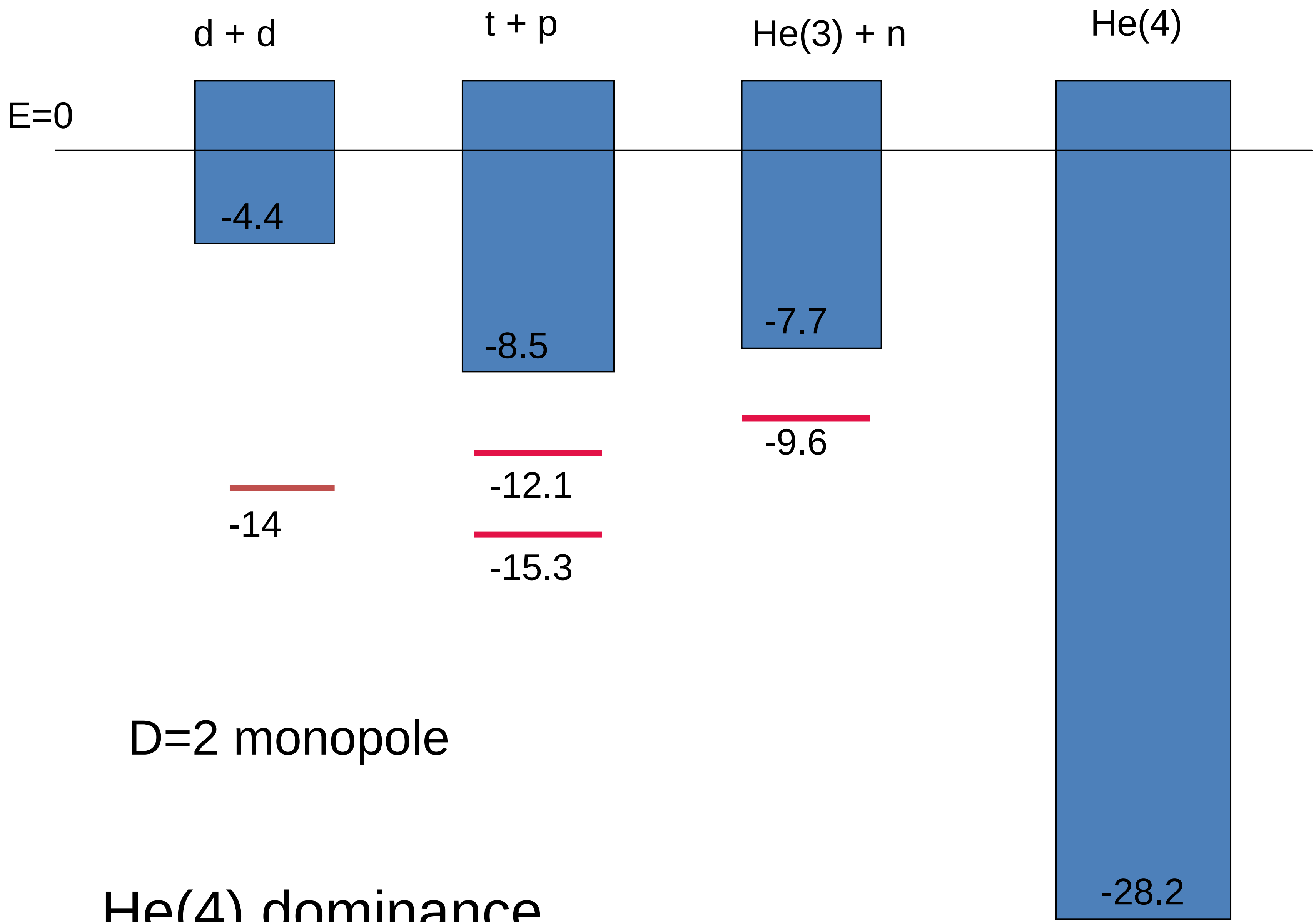
If we start from the ground state of d-d-*e system, namely from $\vec{c}(0) = (1,0,0,0)$, other components increase with time. Since the fourth state ^{4}He-*e does not have bound state, the monopole must emit an α -particle, whose probability is proportional to $|c_4(t)|^2$.

To examine the possibility of other final states (t, p) or (^{3}He, n), it is convenient to draw the graph of the energy spectra of these four states.



D=1 monopole

He(4) dominance



D=2 monopole

He(4) dominance

Energy spectra of A=4 states

- In the graph, the origin E=0 is the energy level when all the nucleons and the monopole separate infinitely.
- The binding energies of nuclei (in vacuum) are 2.2, 8.4, 7.7 and 28.3MeV. for d , t, ^{3}He and ^{4}He respectively. If we use these values, the thresholds of the continuous spectra are E=-4.4, -8.4, -7.7 and -28.3MeV. respectively for |d, d, *e>, It, p, *e>, | ^{3}He, n, *e> and | ^{4}He ,*e> states.

- Since the binding energy of the deuteron with a monopole is around 2MeV., the energy level of the ground state of (d-*e) is E=-4.2MeV..
- When two deuterons are trapped by the monopole, energy level of the ground state is E=-8.4MeV.. From the energy conservation it can transit to ^{4}He , however it cannot become (^{3}He, n) state. (t, p) state is marginal, because the threshold is very close to the starting energy. Even when the threshold is lower, probability of appearance of (t, p) is very small, because of the small phase volume.

Penetration factor P

- When the initial energy is close to zero, two deuterons cannot come to the fm. region to start the nuclear reaction, because of the Coulomb repulsion. In fact, if we compute the penetration factor P by using the WKB approximation at E=0:

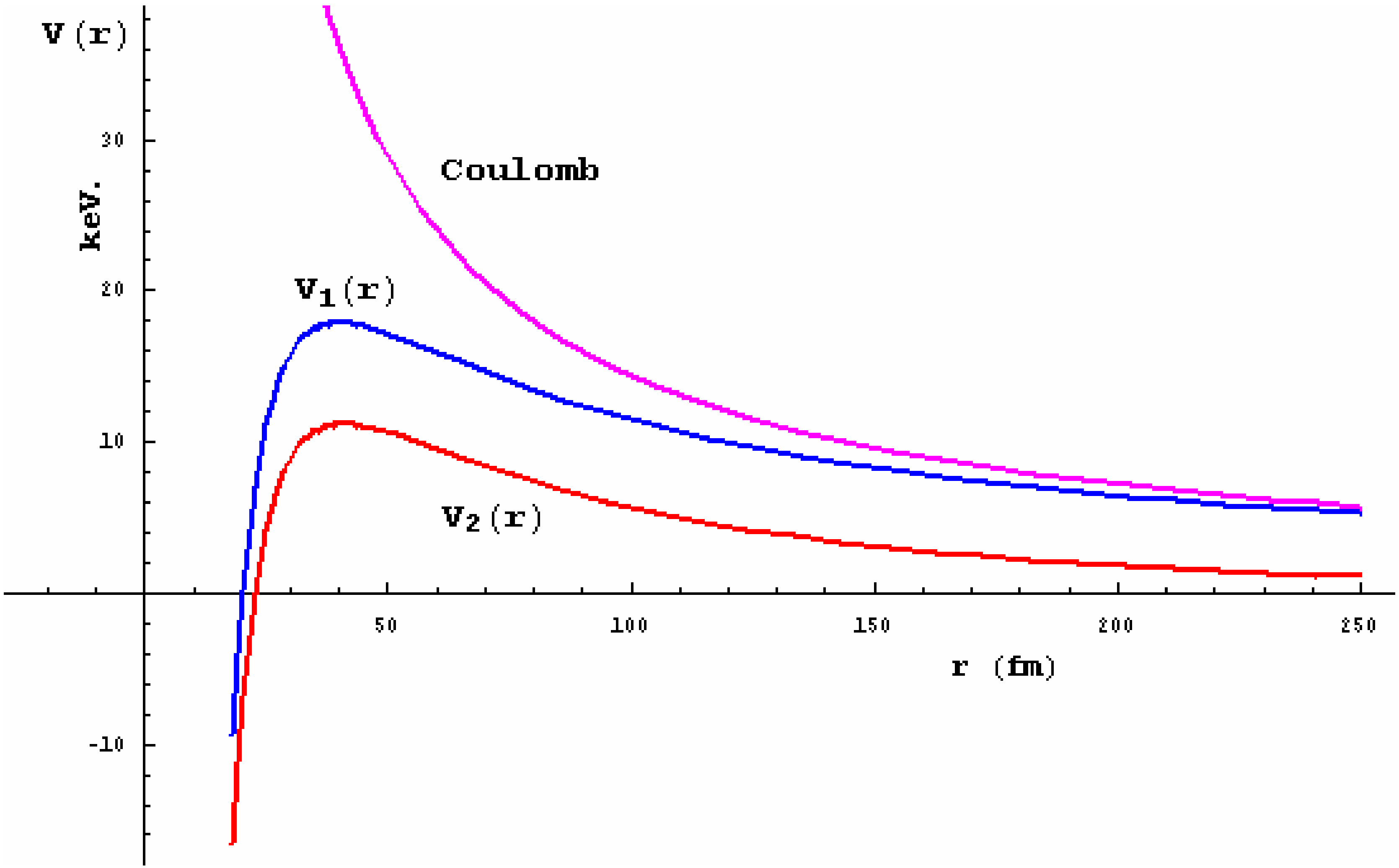
$$P = e^{-2\tau} \quad with \quad \tau = \sqrt{2m_{red}} \int_a^b \sqrt{V(x)}\, dx$$

  , where [a, b] is the domain of the penetration, P of the d + d reaction in vacuum is forbiddingly small: $P = 5.3 \times 10^{-106}$.

- On the other hand, when there is the mediation of the monopole, two deuterons can come close, because of the strong attraction from the monopole. The potential $V_1(r)$ of the second deuteron, when the first deuteron is trapped by the monopole, is $V_1(r) = e^2/r - \kappa_{tot}(e\cdot {}^*eD/2m_p)/r^2$.

- In the electron-rich environment the Coulomb term is shielded and the potential $V_2(r) = e^2 \exp[-2m_e r]/r - \kappa_{tot}(e\cdot {}^*eD/2m_p)/r^2$ m_e

Potential of t + (p+ *e) system (D=2)



Comparison of penetration factors P of (t + p) and (d + d) reactions

- In the limit of E$\rightarrow$ 0, the penetration factor P is

$$P = e^{-2\tau} \quad with \quad \tau = \sqrt{2m_{red}} \int_a^b \sqrt{V(x)}\, dx$$

- In vacuum:

	(d + d)	(t + p)
	τ = 121.20	τ = 104.96
	$P = 5.33 \times 10^{-106}$	$P = 6.78 \times 10^{-92}$

- With the mediation of the magnetic monopole: (for V=V_2)

	[D = 1]	[D = 2]
	$P = 1.22 \times 10^{-4}$	$P = 8.03 \times 10^{-3}$
(*e-t) + p	τ = 4.51	τ = 2.41
	$P = 2.98 \times 10^{-7}$	$P = 2.72 \times 10^{-4}$
(*e-p) + t	τ = 7.51	τ = 4.11
	$P = 6.22 \times 10^{-9}$	$P = 1.42 \times 10^{-7}$
(*e-d) + d	τ =9.45	τ = 7.88

On the non-reproducibility of the nuclear cold fusion

- We learned the information, that the reaction d + d$\rightarrow$ ^{4}He exists, made it possible to narrow down the underlying mechanism of the cold fusion.
- However there is another important information of the cold fusion, that the reaction starts in the sporadic way, namely not on demand.
- It is the general belief of scientists that the fundamental law of Nature must be independent of time and position, in fact the Lagrangian or the Hamiltonian does not involve **x** or t explicitly. However this does not necessarily mean the reaction proceeds on demand.
- As an example, let us consider the case of " one-particle catalyst ", in which a rare particle plays the roll of catalyst. The reaction proceeds only when the catalyst particle comes into and stops in the domain of reaction. In this way, the probability comes into the scene.
- Since the action of the catalyst is independent of time and position, the natural law has the reproducibility. However as a phenomenon, all what we can expect is to observe the correlation between the existence of the catalyst particle and the reaction to proceed by measuring the excess heat for example.

D=1

	q	j (type)	n	$-E$	$\sqrt{<r^2>}$
n	0	1/2 (A)	1	37.37 eV.	647.7 fm
			2	1.375×10^{-6} eV.	8.57×10^{6} fm
			$\vdots$		
			∞	(C_∞ =37.37 eV. ,	$\mu = 0.3670$)
p	1/2	0 (B)	1	0.1882 MeV.	11.00 fm
			2	76.046 eV.	547.6 fm
			3	0.03069 eV.	27257. fm
			$\vdots$		
			∞	(C_∞ =0.1884 MeV. ,	$\mu = 0.8040$)
t	1/2	0 (B)	1	1.516 MeV.	3.820 fm
			2	58.085 keV.	19.36 fm
			3	2.226 keV.	98.89 fm
			$\vdots$		
			∞	(C_∞ =1.516 MeV. ,	$\mu = 1.9263$)
t	1/2	1 (A)	1	2.178 keV.	79.19 fm
			2	24.366 eV.	748.7 fm
			3	0.02725 eV.	7079. fm
			$\vdots$		
			∞	(C_∞ =2.1783 keV. ,	$\mu = 1.3984$)
^3He	1	1/2 (B)	1	0.2454 MeV.	7.371 fm
			2	2.7413 keV.	70.36 fm
			3	30.047 eV.	672.15 fm
			$\vdots$		
			∞	(C_∞ =0.2502 MeV. ,	$\mu = 1.3921$)
d	1/2	1/2 (A)		(no bound states ,	$\mu = -i\ 0.360$)

D=2

	q	j (type)	n	$-E$	$\sqrt{< r^2 >}$
n	0	1/2 (A)	1	0.8003 MeV.	5.728 fm
			2	1.115 keV.	153.97 fm
			3	1.542 eV.	4139.3 fm
			⋮		
			∞	(C_∞ =0.8040 McV. ,	$\mu = 0.9545$)
p	1	1/2 (B)	1	2.4065 MeV.	3.666 fm
			2	15.457 keV.	47.70 fm
			3	98.231 eV.	598.42 fm
			⋮		
			∞	(C_∞ =2.4322 MeV. ,	$\mu = 1.2421$)
t	1	1/2 (B)	1	4.366 MeV.	2.779 fm
			2	0.5479 MeV.	8.464 fm
			3	57.766 keV.	26.31 fm
			⋮		
			∞	(C_∞ =5.4085 MeV. ,	$\mu = 2.7697$)
t	1	3/2 (A)	1	1.203 McV.	4.651 fm
			2	73.162 keV.	19.20 fm
			3	4.2423 keV.	79.88 fm
			⋮		

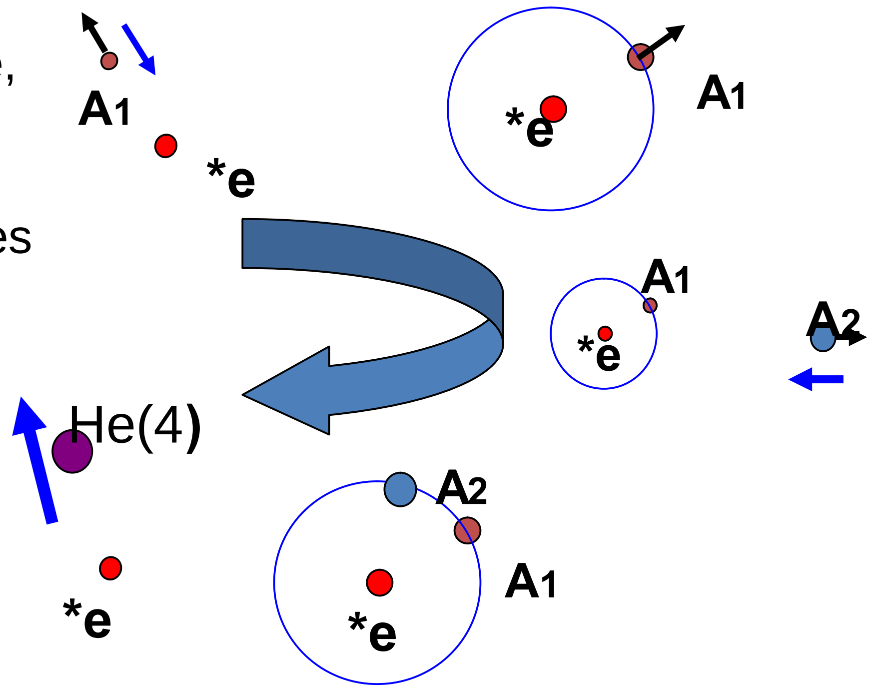
			⋮		
			∞	(C_∞ =5.4085 MeV. ,	$\mu = 2.7697$)
t	1	3/2 (A)	1	1.203 MeV.	4.651 fm
			2	73.162 keV.	19.20 fm
			3	4.2423 keV.	79.88 fm
			⋮		
			∞	(C_∞ =1.2696 MeV. ,	$\mu = 2.2042$)
t	1	5/2 (A)	1	0.5342 eV.	3169.1 fm
			2	8.465×10^{-8} eV.	7.9610×10^{6} fm
			⋮		
			∞	(C_∞ =0.53418 eV. ,	$\mu = 0.4013$)
^3He	2	3/2 (B)	1	1.063 MeV.	4.596 fm
			2	51.115 keV.	21.50 fm
			3	2.3239 keV.	100.9 fm
			⋮		
			∞	(C_∞ =1.1259 MeV. ,	$\mu = 2.0312$)
d	1	0 (B)	1	508.205 eV.	152.8 fm
			2	0.33595 eV.	5910.0 fm
			⋮		
			∞	(C_∞ =575.970 eV. ,	$\mu = 0.8450$)

An order of magnitude estimation of the rate of the energy production (just for fun)

Let us start by choosing the natural unit that c=1 and $\hbar$=1. To make all the quantitie dimensionless, we need the unit of length, we shall choose it as the Compton wave length of the charged pion $\ell_0 = \hbar / m_{\pi^+} c = 1.41 fm.$, because we are checking the wo of strong interaction. Then the unit of time is $t_0 = \ell_0 / c = 4.7 \times 10^{-24}$ sec..

The neck point of the fusion cycle is the penetration of the second nucleus, whose value is $P = 6.22 \times 10^{-9}$ for the Dirac monopole (D=1). The speed of the cycle must be reduced by this factor. So the number of cycle per 1 sec. is $n = P$ (1 sec.)$/t_0$, and which is $n = 1.3 \times 10^{15}$.

If we remember 1eV.=1.60×10^{-19} Joule, the energy production per cycle is $E_0 =$ 23.8MeV.=3.81×10^{-12}J. Therefore the total energy production per sec. becomes $n E_0 = 5.0 \times 10^3$Joule. So the energy production rate is 5kW..



On the other hand, for the case of the Schwinger monopole(D=2), the penetration factor is $P = 1.4 \times 10^{-7}$, so $n E_0 = 1.13 \times 10^5$ Joule. Therefore