A prototype of the nuclear cold fusion reaction discovered by theoretical particle physicist

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There are two different types of scientists who believe in the reality of the nuclear cold fusion. The researchers, who observed the excess energy by experiments, belong to the first type. On the other hand, a small number of theoreticians, who are working on the physics of the magnetic monopole, know that the nuclear reaction of the zero incident energy proceeds when the system involves a magnetic monopole. The purpose of this talk is to explain to the former group how the theoretician of the particle physics comes to arrive at the conclusion that the nuclear cold fusion must occur if a magnetic monopole exists, in the framework of the quantum theory.

first group
(chemist, engineer, physicist ...)

[tools]
calorimeter,
mass spectrometer

[starting year]
1989

[motivation]
Possibility of high density absorption of hydrogen isotope by Pd.

[style of theory]
Mostly model making,
Predictive power is not high.

second group
(particle physicist, theory)

[tools]
quantum theory

.

[starting year]
1983

[motivation]
Contamination of the magnetic monopole by the space dust

[style of theory]
Giving hamiltonian then solve.
Predictive power is high, because of the neutrality in derivation.

Let us cosider a container filled with D_2 gas of 1 atm. and keep its temperature around 300° K. Everybody know that it remains unchanged, in particular no nuclear reaction proceeds.

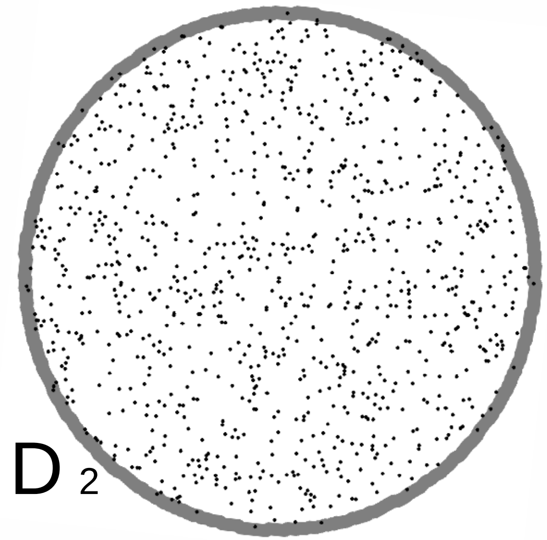
The standard explanation of this fact is done by the calculation of the penetration factor P by $P = \exp[-2\tau]$ where

$$\tau = \int_a^b \sqrt{2m_{red}(V(x) - E)}dx \quad ,$$

in which the WKB (Wenzel, Kramers and Brillouin) approximatin is used.[fp.(15)] For the Coulomb potential $V(x) = e^2/x$ and in the limit $E \to 0$, τ becomes

$$\tau = 2e\sqrt{2m_{red}}(\sqrt{b} - \sqrt{a}) \quad .$$

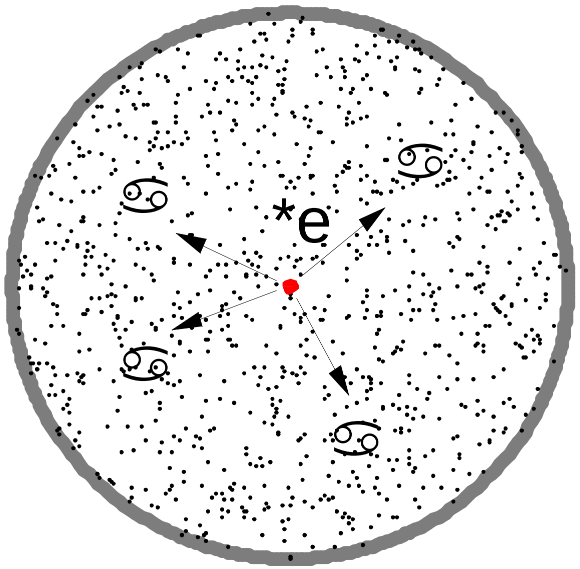
If we put in all the necessary numbers, $e^2 = 1/137$, $m_{red} = 6.73$, $\sqrt{b} = 193.7$ and $\sqrt{a} = 1.0$, then $\tau = 120.8$, which means the penetration factor is $P = 2.2 \times 10^{-106}$.



On the other hand, if we add a magnetic monopole to the D_2 system, and fix it in the container, then the monopole starts to attract surrounding deuterons and to emit the α-particle. This process continues until the magnetic monopole leaves the container. The purpose of this talk is to explain this phenomenon in the framework of the quantum theory.

The secret of this process is the possiblity for the magnetic monopole to attract the nuclei with the magnetic moment such as p, d, t and ^3He, when the tail of the magnetic moment orients to the direction of the magnetic monopole. On the other hand the spin-0 particl such as ^4He is repeled by the monopole.

The attractive force between the monopole and the fuel neucleus is strong enough to form the bound state whose orbital size is several fm.. When two deuterons are trapped by the monopole, they quickly fuse to become the tighter nucleus ^4He. However it must leave the monopole, and so there remains a fresh monopole, and it starts to attract the sorounding deuterons again. This means the monopole plays the roll of the catalyzer of the nuclear fusion.



[Plan of the talk]

1. Brief review of the theory of the magnetic monopole.
2. To solve the three eigen-value problems necessary for the cold fusion
   - (a) system of monopole-spin 0 charged particle (Tamm, 1931)
   - (b) system of monopole-nucleon (ie. spin 1/2) (T.S. , 1983)
   - (c) system of momopole-electron (K-Y , 1977)
3. Nuclear cold fusion reactor
4. Comparison with experiments
   - (a) possibility of the nuclear reaction with very low incident energy
   - (b) ^4He dominance in d+d reaction
5. Problem of the inverse proposition (monopole $\rightleftharpoons$ cold fusion)

Charge quantization condition of Dirac

[Maxwell equations in vacuum]

The Maxwell equations without charge and current are

$$\begin{aligned} div\vec{D} &= 0 \\ div\vec{B} &= 0 \end{aligned}$$

and

$$\begin{aligned} rot\vec{H} - \frac{1}{c}\frac{\partial \vec{D}}{\partial t} &= 0 \\ rot\vec{E} + \frac{1}{c}\frac{\partial \vec{B}}{\partial t} &= 0. \end{aligned}$$

We can see that $\vec{D}$ and $\vec{B}$ (and also $\vec{E}$ and $\vec{H}$) appear in the symmetrical way in Maxwell equations in vacuum. Or more precisely, the duality transformation

$$\begin{aligned} \vec{D} &\rightarrow \vec{B} \quad and \quad \vec{E} \rightarrow \vec{H} \\ \vec{B} &\rightarrow -\vec{D} \quad and \quad \vec{H} \rightarrow -\vec{E} \end{aligned}$$

, which interchanges the electric and the magnetic objects, does not change the Maxwell equations in vacuum (duality invariance).

[The Maxwell equations]

The Maxwell equations are

$$\begin{aligned} div\vec{D} &= 4\pi\rho \\ div\vec{B} &= 0 \end{aligned}$$

and

$$\begin{aligned} rot\vec{H} - \frac{1}{c}\frac{\partial\vec{D}}{\partial t} &= \frac{4\pi}{c}\vec{i} \\ rot\vec{E} + \frac{1}{c}\frac{\partial\vec{B}}{\partial t} &= 0. \end{aligned}$$

The invariance under the duality transformation is destroyed, and the electric object and the magnetic object are not treated on the same footing. For example, although $\vec{D}$ has source ρ, $\vec{B}$ does not have.

[Maxwell equations with dual symmetry]

The Maxwell equations modified by Dirac are

$$\begin{aligned} div\vec{D} &= 4\pi\rho \\ div\vec{B} &= 4\pi\,{}^{\star}\rho \end{aligned}$$

and

$$\begin{aligned} rot\vec{H} - \frac{1}{c}\frac{\partial\vec{D}}{\partial t} &= \frac{4\pi}{c}\vec{i} \\ rot\vec{E} + \frac{1}{c}\frac{\partial\vec{B}}{\partial t} &= -\frac{4\pi}{c}\,{}^{\star}\vec{i}. \end{aligned}$$

The duality transformation including the charge and current

$$\begin{aligned} \vec{D} &\rightarrow \vec{B} \\ \vec{B} &\rightarrow -\vec{D} \qquad etc. \end{aligned}$$

and

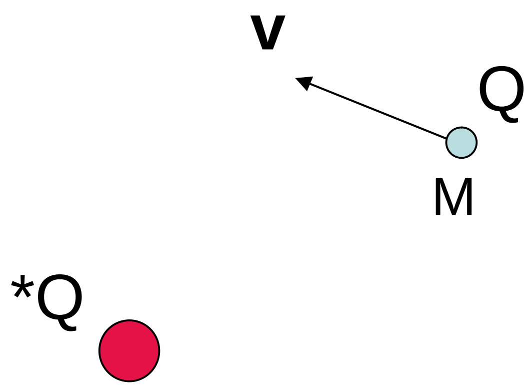
$$\begin{aligned} \rho &\rightarrow {}^{\star}\rho \\ {}^{\star}\rho &\rightarrow -\rho \qquad etc. \end{aligned}$$

, which interchanges the electric and the magnetic objects, does not change the modified Maxwell equations. The duality invariance is recovered.

[Extra angular momentum of the charge-monopole system]

In order to see the the existence of the angular momentum $^*QQ/c$, let us cosider monopole with magnetic charge $^*Q$ is fixed at the origin, and a particle of mass M and electric charge Q is moving in the magnetic Coulomb field produced by the monopole. The equation of motion is

$$M\ddot{\vec{r}} = (Q/c)\dot{\vec{r}} \times \vec{B} \qquad \vec{B} = {}^*Q\frac{\vec{r}}{r^3} \quad ,$$



and as in the case to prove the conservation of the angular momentum, we make the vector product $\vec{r}\times$ of the equation of motion

$$M\frac{d}{dt}(\vec{r} \times \dot{\vec{r}}) = (\frac{^*QQ}{c})\frac{\vec{r} \times (\dot{\vec{r}} \times \vec{r})}{r^3} \quad .$$

From the formula [fp (1)], we obtain

$$M\frac{d}{dt}(\vec{r} \times \dot{\vec{r}}) = (\frac{^*QQ}{c})(\frac{\dot{\vec{r}}}{r} - \frac{\vec{r}(\vec{r} \cdot \dot{\vec{r}})}{r^3}) = (\frac{^*QQ}{c})\frac{d}{dt}(\frac{\vec{r}}{r}) \quad .$$

The same extra angular momentum is also obtaind if we integrate the angular momentum density of the electro-magnetic field $\vec{r'} \times \epsilon\mu\vec{S}(\vec{r'})$ stored in space, in which $\vec{S}$ is Poynting vector.

Therefore what is conserved in time is

$$\vec{L} = M\vec{r} \times \dot{\vec{r}} - (^*QQ/c)\hat{r} \quad .$$

[Charge quantization condition]

Since in quantum mechanics a component of the angular momentum can assume only the integer multiple of $\hbar/2$, we can derive the charge quantization condition of Dirac

$$\frac{{}^{*}QQ}{c} = \frac{\hbar}{2}n \qquad n = 0, \pm 1, \pm 2 \cdots \qquad (Dirac).$$

On the other hand, Schwinger claims that the component of the extra angular momentum assumes not $\hbar n/2$ but $\hbar n$, because the term $-({}^{*}QQ/c)\hat{r}$ is obtained classically.

Since we are going to consider both cases, let us introduce a magnetic charge number D and assign $D = 1$ for Dirac and $D = 2$ for Schwinger, and write the charge quantization as

$$\frac{{}^{*}QQ}{\hbar c} = \frac{D}{2}n \qquad n = 0, \pm 1, \pm 2 \cdots$$

If we write the non-zero smallest magnetic charge as ${}^{*}e$, the electric charge Q becomes

$$Q = \frac{\hbar c}{{}^{*}e}\frac{D}{2}n \qquad n = 0, \pm 1, \pm 2 \cdots .$$

This equation indicate that the electric charge Q is discrete and is integer multiple of $(\hbar c D/2{}^{*}e)$. Moreover we can understand the equality of the electric charges of the electron and proton up to the sign on the basis of the quantum theory, on the other hand experimentally the equality has been known with extremely high relative accuracy 10^{-22}.

["Fine structure constant" of the magnetic charge]

For the non-vanishing smallest charges e and $^{\star}e$, the charge quantization condition becomes ($n = 1$)

$$\frac{^{\star}ee}{\hbar c} = \frac{D}{2} \quad and \quad so \quad \frac{^{\star}e^2}{\hbar c}\frac{e^2}{\hbar c} = \frac{D^2}{4}$$

Since the numerical value of the fine structure constant is $e^2/\hbar c = 1/137.036$, the magnetic counterpart of the "fine structure constant" is determined:

$$\frac{^{\star}e^2}{\hbar c} = 137.036\frac{D^2}{4} \quad .$$

Therefore the Coulomb force between the magnetic monopole is super-strong. Schwinger used the super-strong Coulomb force to construct the dyon model of hadron.

Since the magnetic Coulomb field is super-strong, we may expect the magnetic moment of a nucleus is attracted to the monopole and forms the bound states. This is in fact the case, since the potential of such an interaction is

$$V(r) = -\kappa_{tot}(\frac{e}{2m_p})\vec{\sigma}\cdot\vec{B}(r) \qquad with \quad \vec{B} = {^{\star}e}\frac{\hat{r}}{r^2} \quad .$$

From the charge quantization condition $^{\star}ee = 1/2$,

$$V(r) = -\kappa_{tot}(\frac{D}{4m_p})\frac{(\vec{\sigma}\cdot\hat{r})}{r^2}$$

The strength of this potential $V(r)$ has the same order of magnitude as that of the nuclear potential. For example, for proton the potentials $V(r)$ are -9.3 MeV. and -2.33 MeV. at $r = 1.4fm$ and at $r = 2.8fm$ respectively (for D=1).

[The vector potential of magnetic Coulomb field]

The standard prescription to include the effect of the magnetic field $\vec{B}$ in the quantum calculation is to make the substitution $-i\vec{\nabla} \longrightarrow -i\vec{\nabla} - (Ze/c)\vec{A}$, where *vecA* is the vector potential whose rotation is $\vec{B}$. In particular for the magnetic Coulomb field namely for $\vec{B} =^\star Q\vec{r}/r^3$. The rotation of the vector potential

$$A_r = 0, \quad A_\theta = 0 \quad and \quad A_\phi =^\star Q\frac{(c - \cos\theta)}{r\sin\theta}$$

is the Coulomb field as we can check by using the formula [fp.(2)]. In the equation c is an arbitrary constant. However this $\vec{A}$ has singularity at $\sin\theta = 0$ namely on the whole z-axis. If $c = 1$, the singularity on $z > 0$ part disappears, similarly if $c = -1$, the singularity on $z < 0$ part disappears. Therefore in order to avoid the singularity of $\vec{A}$, we divide the whole domain into two part, for example the north hemisphere and the south hemispere with small overlapping region around equator, and assign $\vec{A}$ with suitable c.

The standard choice is:

$$\vec{A}^{(N)} = {}^\star Q\frac{(1 - \cos\theta)}{r\sin\theta}\hat{\phi} \qquad (north)$$

and

$$\vec{A}^{(S)} = -{}^\star Q\frac{(1 + \cos\theta)}{r\sin\theta}\hat{\phi} \qquad (south)$$

On the overlapping region around the equator, these potentials are related by the gauge transformation, because

$$\vec{A}^{(S)} - \vec{A}^{(N)} = -(\frac{2\ {}^\star e}{r\sin\theta})\hat{\phi} = \nabla(-2\ {}^\star e\phi) \quad ,$$

and both vector potentials describe the same physical entity.

[The monopole harmonics $Y_{q,\ell,m}(\theta,\phi)$]

If $\vec{L} = M\vec{r}\times\dot{\vec{r}} - q\hat{r}$, the eigen-function of $\vec{L}^2$ and L_z is the monopole harmonics $Y_{q,\ell,m}(\theta,\phi)$, and which reduces to the ordinary spherical harmonics $Y_{\ell,m}(\theta,\phi)$ when $q=0$. If we write $Y_{q,\ell,m}(\theta,\phi) = e^{\pm iq\phi}e^{im\phi}\Theta(\theta)$, where $\pm$ of the exponential corresponds to the nothern and the southern hemisphere respectively, $\Theta(z)$ must satisfy

$$[\ell(\ell+1)-q^2]\Theta = -(1-z^2)\Theta'' + 2z\Theta' + \frac{(m+qz)^2}{1-z^2}\Theta$$

The explicit form of $Y_{q,\ell,m}(\theta,\phi)$ is

$$Y^{(n)}_{q,\ell,m}(\theta,\phi) = \sqrt{\frac{2\ell+1}{4\pi}}e^{+iq\phi}d^{(\ell)}_{-q,m}(\theta)e^{+im\phi} \qquad in \quad R_n$$

and

$$Y^{(s)}_{q,\ell,m}(\theta,\phi) = e^{-2iq\phi}Y^{(n)}_{q,\ell,m}(\theta,\phi) \qquad in \quad R_s,$$

where R_n and R_s is the northern and southern hemispheres of the sphere respectively, and $d^{(\ell)}_{m',m}(\theta)$ is Wigner's d-function of rotation which is widely used in the nuclear physics. [fp. (8)]

For details, visit the home page of the Research Institute of Magnetic Monopole:
http://www.fureai.or.jp/~t-sawada/
and
click: fundamentals of magnetic monopole.

Hamiltonians of three eigen-value problems

[1.] Charged spin-0 particle in the magnetic Coulomb field.

$$H_0 = \frac{1}{2M_A}(-i\vec{\nabla} - Ze\vec{A})^2$$

.

[2.] Nucleon in the magnetic Coulomb field.

$$H = \frac{1}{2M_A}(-i\vec{\nabla} - Ze\vec{A})^2 - \kappa_{tot} D \frac{^*ee}{2m_p} \frac{(\vec{\sigma} \cdot \hat{r})}{r^2} F(r)$$

, where $F(r)$ comes from the nucleon form factor:

$$F(r) = (1 - e^{-ar}(1 + ar + \frac{a^2r^2}{2})) \quad with \quad a = 6.04\mu_\pi \quad .$$

[3] Dirac electron in the magnetic Coulomb field.

$$H_D = \vec{\alpha} \cdot (\vec{p} - Ze\vec{A}) + \beta M - \frac{\kappa_a q}{2Mr^2} \beta \begin{bmatrix} (\vec{\sigma} \cdot \hat{r}) & 0 \\ 0 & (\vec{\sigma} \cdot \hat{r}) \end{bmatrix}$$

where the Dirac matrices $\vec{\alpha}$ and β are

$$\vec{\alpha} = \begin{bmatrix} 0 & \vec{\sigma} \\ \vec{\sigma} & 0 \end{bmatrix} \quad and \quad \beta = \begin{bmatrix} I_2 & 0 \\ 0 & -I_2 \end{bmatrix}$$

Eigen-state of the spin-0 particle

For spin-0 particle such as ^4He the equation of $R(r)$ becomes

$$[-\frac{1}{2m_A r^2}\frac{d}{dr}(r^2\frac{d}{dr}) + \frac{\ell(\ell+1)-q^2}{2m_A r^2} - E] \quad R(r) = 0 \quad ,$$

where the radial function R is

$$\psi = R(r)Y_{q,\ell,m} \quad with \quad \ell = |q|, \quad |q|+1, \quad |q|+2\cdots$$

The solution, which does not blow up at $r=0$, is [fp.(5)]

$$R(r) = \frac{1}{\sqrt{kr}}J_\mu(kr) \quad ,$$

where

$$\mu = \sqrt{\ell(\ell+1)-q^2+1/4} = \sqrt{(\ell+1/2)^2-q^2} > 0 \quad and \quad k = \sqrt{2m_A E} \quad .$$

For $E < 0$, there is no meaningful solution, and therefore no bound state.

Even for the smallest ℓ namely for $\ell = |q|$, the behavior of $R(r)$ at $r=0$ is [fp.(5')]

$$R(r) \approx c_1(kr)^{\sqrt{|q|+1/4}-1/2}$$

, therefore the wave function is repeled from the origin unless $q=0$.

Eigen-state of the spin-1/2 particle

[Angular function of spin 1/2 particle in the monopole field]

For the spin-1/2 particle, we can construct the state of the total angular momentum (j, m) by combining spin-up and spin-down states with the monopole harmonics $Y_{q,\ell,m'}$, in which the Clebsch-Gordan coefficients appear.[fp.(6)] In general, for given j there are two states

$$
\begin{aligned}
\Phi^{(1)}_{j,m} &= \sqrt{\frac{j+m}{2j}} Y_{q,j-1/2,m-1/2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \sqrt{\frac{j-m}{2j}} Y_{q,j-1/2,m+1/2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} \sqrt{\frac{j+m}{2j}} Y_{q,j-1/2,m-1/2} \\ \sqrt{\frac{j-m}{2j}} Y_{q,j-1/2,m+1/2} \end{bmatrix}
\end{aligned}
$$

and

$$
\begin{aligned}
\Phi^{(2)}_{j,m} &= -\sqrt{\frac{j-m+1}{2j+2}} Y_{q,j+1/2,m-1/2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \sqrt{\frac{j+m+1}{2j+2}} Y_{q,j+1/2,m+1/2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} \\
&= \begin{bmatrix} -\sqrt{\frac{j-m+1}{2j+2}} Y_{q,j+1/2,m-1/2} \\ \sqrt{\frac{j+m+1}{2j+2}} Y_{q,j+1/2,m+1/2} \end{bmatrix}
\end{aligned}
$$

except for the smallest j, namely for $j = |q| - 1/2$. After Yang we shall call the (j, m) state with $j \geq |q| + 1/2$ type-A, to which two states $\Phi^{(1)}_{j,m}$ and $\Phi^{(2)}_{j,m}$ belong. On the other hand state of $j = |q| - 1/2$ is the type-B and to which only $\Phi^{(2)}_{j,m}$ belongs.

"hedgehog" state of the spin-angular function

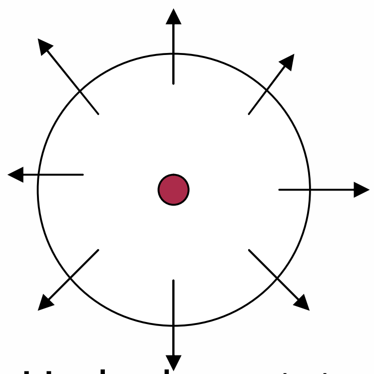
In general, the ground state appears in the smallest j state, namely in $j = |q| - 1/2$, which has the form

$$\eta_m = \Phi^{(2)}_{|q|-1/2,m} = \begin{bmatrix} -\sqrt{\frac{|q|-m+1/2}{2|q|+1}} Y_{q,|q|,m-1/2} \\ \sqrt{\frac{|q|+m+1/2}{2|q|+1}} Y_{q,|q|,m+1/2} \end{bmatrix} \quad with \quad -j \le m \le j \quad .$$

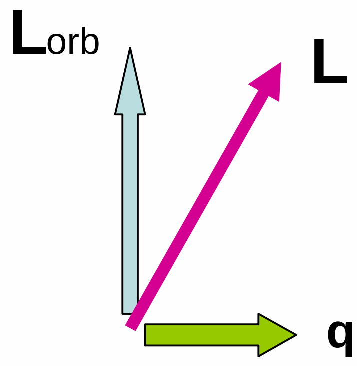
This type-B state η_m is the eigen-state of the (pseudo-)scalar operator $(\vec{\sigma} \cdot \hat{r})$, in fact

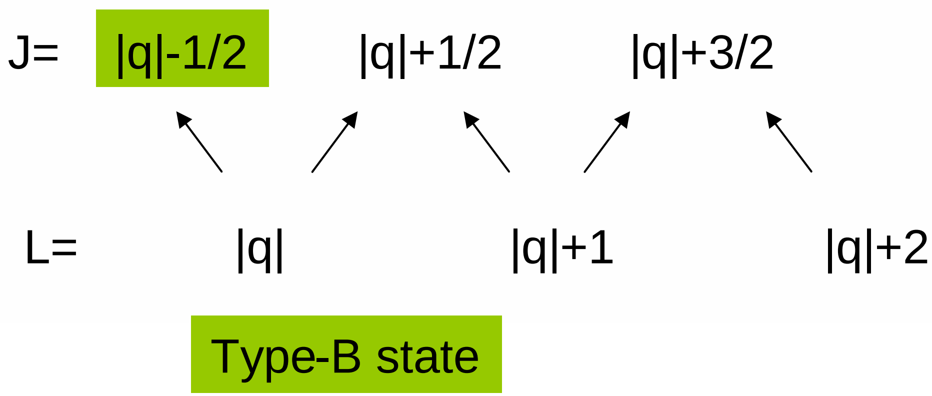
$$(\vec{\sigma} \cdot \hat{r})\eta_m = \frac{q}{|q|}\eta_m \quad .$$

As it is shown in the figure, the spin orients to outward at all the places, so it is named "hedgehog" state, and therefore the magnetic moment of the spin is attracted most strongly to the direction of the monopole, which is fixed at the origin.



Hedgehog state





[Example of the eigenvalue problem of spin-1/2 particle (type-B) 1]

The equation to be solved is

$$[\frac{1}{2M}(-i\vec{\nabla} - Ze\vec{A})^2 - \frac{\kappa_{tot}q}{2M_p r^2}(\vec{\sigma}\cdot\hat{r})F(r)]\psi = E\psi$$

, where

$$F(r) = (1 - e^{-ar}(1 + ar + \frac{a^2r^2}{2}))$$

with

$$q = ZD/2 \quad and \quad a = 6.04\mu_\pi.$$

If we remember

$$\vec{L} = \vec{r} \times (\vec{p} - Ze\vec{A}) - q\hat{r}$$

and

$$(\vec{p} - Ze\vec{A})^2 = -\frac{1}{r^2}\frac{d}{dr}(r^2\frac{d}{dr}) + \frac{1}{r^2}(\vec{L}^2 - q^2)$$

, for the type-B amplitude

$$\psi = \frac{1}{r}g(r)\eta_{bm}$$

the Schrödinger equation of the radial function for the proton becomes

$$[\frac{d^2}{dr^2} - \kappa^2 - \frac{|q| - \kappa_{tot}qF(r)}{r^2}]\, g(r) = 0$$

with $\kappa^2 = -2M_pE$.

[Example of the eigenvalue problem of spin-1/2 particle (type-B) 2]

It is not difficult to find the eigen-values and the eigen-function with the boundary conditions that $g(0) = 0$ and $g(r)$ damps for large r. It is convenient to know that for $ar \gg 1$, $F(r) \to 1$ and so the differential equation reduces to the modified Bessel equation.[fp.(5)] Therefore for large ar, the damping solution is the K-Bessel:

$$g(r) = \sqrt{\kappa r} K_{i\mu}(\kappa r) \quad with i\mu = \sqrt{|q| - \kappa_{tot} q + 1/4} \quad .$$

We must join the solution of the lower r region smoothly to the asymptotic solution given above at the matching point $r = r_m$, where $ar_m \gg 1$. However since $K_{i\mu}(\kappa r_m)$ is a oscillatory function of κ, there appear infinitely many bound states, which are designated by the principal quantum number n. The list is the energy level and the size of the orbit of the monopole-proton system.

D=1 ($\mu = 0.8040$)

$-E_1 = 0.1879$MeV. $\quad \bar{r_1} = 11.00$fm.

$-E_2 = 75.91$eV. $\quad \bar{r_1} = 547.0$fm.

in general,

$$-E_n = 0.1879 e^{-2\pi(n-1)/\mu} \text{ MeV.}$$

D=2 ($\mu = 1.2422$)

$-E_1 = 2.428$MeV. $\quad \bar{r_1} = 3.67$fm.

$-E_2 = 15.43$keV. $\quad \bar{r_1} = 47.7$fm.

$-E_3 = 98.11$eV. $\quad \bar{r_1} = 598.$fm.

in general,

$$-E_n = 2.428 e^{-2\pi(n-1)/\mu} \text{ MeV.}$$

we can evaluate the size of the bound states by

$$\bar{r} = \sqrt{< r^2 >} = \sqrt{\frac{2}{3}(1 + \mu^2)\frac{1}{(-2ME_n)}}$$

Dirac electron in the external magnetic Coulomb field 1

The hamiltonian written in the matrix form is

$$H = \begin{pmatrix} M & \vec{\sigma}\cdot(-i\vec{\nabla} - Ze\vec{A}) \\ \vec{\sigma}\cdot(-i\vec{\nabla} - Ze\vec{A}) & -M \end{pmatrix} - \frac{\kappa_a q}{2Mr^3}\begin{pmatrix} \vec{\sigma}\cdot\vec{r} & 0 \\ 0 & -\vec{\sigma}\cdot\vec{r} \end{pmatrix}$$

where κ_a is the anomalous magnetic moment of the electron and numerically $\kappa_a = e^2/2\pi = 0.00116$. The last term is known as the Pauli term.

Let us consider the eigen-value equation $H\psi = E'\psi$, where E' is the total energy and the rest mass energy included. We shall look for the wave function of the form:

$$\psi_m^{(B)} = \begin{bmatrix} f(r)\eta_m \\ g(r)\eta_m \end{bmatrix}$$

If we substitute this form in the eigen-value equation, we obtain

$$[M - E' - \kappa_a|q|(2Mr^2)^{-1}]f(r) - iq|q|^{-1}(\partial_r + r^{-1})g(r) = 0$$

and

$$-iq|q|^{-1}(\partial_r + r^{-1})f(r) - [M + E' - \kappa_a|q|(2Mr^2)^{-1}]g(r) = 0$$

Dirac electron in the external magnetic Coulomb field 2

It is convenient to introduce functions $F(r)$ and $G(r)$ by

$$f(r) = \frac{\kappa_a q}{|\kappa_a q|}\frac{F(r)}{r} \quad and \quad g(r) = -i\frac{G(r)}{r}$$

Then $F(r)$ and $G(r)$ must satisfy

$$\begin{aligned}
\frac{dG}{dr} &= [-\frac{(E'-M)\kappa_a}{|\kappa_a|} - \frac{|\kappa_a q|}{2Mr^2}]F(r) \\
\frac{dF}{dr} &= [\frac{(E'+M)\kappa_a}{|\kappa_a|} - \frac{|\kappa_a q|}{2Mr^2}]G(r)
\end{aligned}$$

There is a solution at $E' = 0$

$$E' = 0 \qquad , \qquad F(r) = -G(r) = \exp[-\frac{\kappa_a}{|\kappa_a|}Mr - \frac{|\kappa_a q|}{2Mr}]$$

and it satisfies the required boundary conditions as long as $\kappa_a > 0$. The required boundary conditions are that $F(0) = G(0) = 0$ and that $F(r)$ and $G(r)$ damp exponentially at $r \to \infty$.

The charge density ρ of the electron of this bound state is $\rho = -e\bar{\psi}\beta\psi$. If we take the limit of $\kappa_a \to 0+$,

$$4\pi r^2\rho(r) = \frac{-e}{2M}\exp[-2Mr] \quad .$$

The mean radius of the electron cloud is $\sqrt{< r^2 >} = \sqrt{2}/(2M)$, where M is the electron mass. Numerically $\sqrt{< r^2 >} = 272.3$ fm.. In the electron-rich environment, the magnetic monopole is shielded by the electron cloud with size 300fm., so when a nucleus of $Z = 1$ forms a bound state with the monopole, whose orbital radius is several fm., the nucleus is shielded also. Therefore the second nucleus can approach to the monopole-nucleus bound system rather freely without feeling the Coulomb repulsion until $x = 300$fm.. This shielding along with the attractive force between the monopole and the magnetic moment of the second nucleus improve the penetration factor P appreciably.

There is one remark. In the relativistic calculation, it is customary to include the rest mass in the energy E' such as $E' = \sqrt{\vec{p}^2 + M^2}$. When the force is not acting, continuous spectra appear in $E' \geq M$ and in $E' \leq -M$, the latter is the "negative sea". There is a gap $M \geq E' \geq -M$ where no level exists when the external potential is zero. So the new level $E' = 0$ means the deep bound state.

Contamination of the magnetic monopole in space

If the magnetic monopole arrives to the earth after the long trip in space, it must be contaminated by the space dust, which mainly consists of the proton, and in small portion consist of light nuclei. In 1983, I found infinitely many bound states of the monopole-proton system, by solving the eigen-value problem of this system, and confirmed the possibility of such contamination.

The small nuclei of spin 1/2 such as triton and ^3He can be treated in the same way as in the case of the proton, in the approximation that the deformation of the nucleus in the magnetic Coulomb field is negligible, although we must change the parameters κ_{tot}, m and a, where a relates to the radius of the nucleus r_1 by $a = \sqrt{12}/r_1$. The binding enegy and the size of the ground state of t and ^3He are

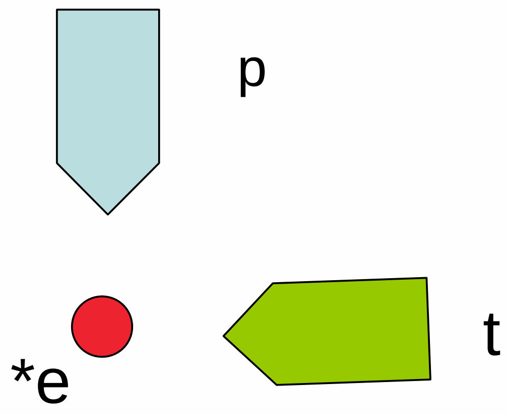
D=1 (triton)	D=2	D=1 (^3He)	D=2
$E = -1.52$ MeV.	$E = -4.37$ MeV	$E = -0.245$ MeV.	$E = -1.06$ MeV
$\bar{r} = 3.82$ fm.	$\bar{r} = 2.78$ fm.	$\bar{r} = 7.37$ fm.	$\bar{r} = 4.60$ fm.

Concerning the deuteron, since the deformation of the nucleus is important, exact calculation of the two-body system in the external potential is required. The binding energy is roughly around -E=1.7MeV. and 4.8 MeV. respectively for D=1 and D=2.

Double contamination of the monopole and the cold fusion

Let us consider case in which two small nuclei are trapped by the monopole $^{*}e$. Hereafter, we shall consider cases $(d -^{*} e - d)$ and $(t -^{*} e - p)$. Since the two nuclei are trapped by the same monopole with the orbital size of several fm., they quickly fuse to become more stable nucleus ^{4}He, in such a process the spin-flip term of the nuclear potential plays the important role. However since the spin of the produced α-particle is zero, from the Tamm's solution, the α cannot stay close to the monopole, but is emitted with energy around 20MeV.. There remains a fresh monopole, and it must attracts fuel nuclei anew.

With such knowledge, we can design the nuclear fusion reactor, which operates at the room temperature.
The figure is such a reactor designed in 1983. A magnetic monopole is fixed firstly, for example by electromagnetic way. The nozzles of proton and the triton jet alternatively and intermittently.



Change of the repulsive Coulomb potential

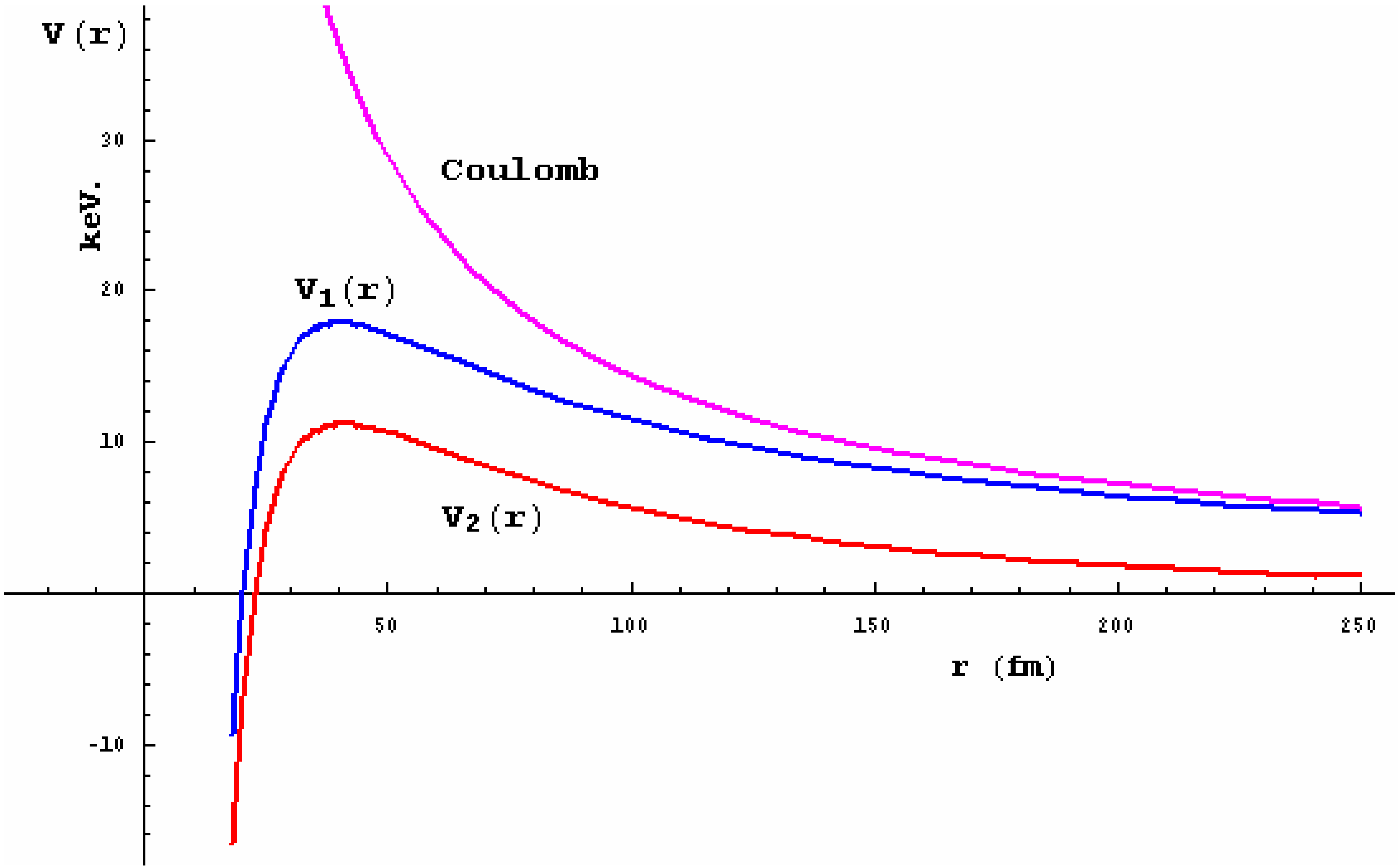
Since the repulsive Coulomb force always exists between the two nuclei, we must consider how the two nuclei can be trapped by the same monopole, notwithstanding the Coulomb repulsion. As an example, let us consider $p+t$ reaction, in which p is trapped by the monopole first and then second nucleus t comes to approach to the system of the bound state $^*e-p$. To the triton t, two forces are acting, one is the Coulomb repulsion and other is the attractive force between the magnetic monopole and the magnetic moment of the triton if the spin of t orients to the opposite direction to the monopole. Therefore the potential of the penetration $V_1(x)$ is

$$V_1(x) = \frac{e^2}{x} - \kappa_{tot}\frac{1}{2m_p}\frac{D}{2}\frac{1}{x^2} \quad .$$

Moreover if the charge of the proton is shielded by the electron cloud of the size $1/(2m_e)$, the potential of the penetration becomes

$$V_2(x) = \frac{e^2}{x}\exp[-2m_e x] - \kappa_{tot}\frac{1}{2m_p}\frac{D}{2}\frac{1}{x^2} \quad .$$

Potential of t + (p+ *e) system (D=2)



Penetration factor P of $p + t$ and $d + d$ systems

For the limit of the zero incident energy $E \to 0$, we use

$$P = e^{-2\tau} \quad with \quad \tau = \sqrt{2m_{red}} \int_a^b \sqrt{V(x)} dx \quad .$$

in vacuum

(d+d)
$\tau = 121.20$
$P = 5.33 \times 10^{-106}$

(t+p)
$\tau = 104.96$
$P = 6.78 \times 10^{-92}$

with the help of the magnetic monopole

[D=1]

- $({}^{*}e - t) + p$
  $\tau = 4.51$
  $P = 1.22 \times 10^{-4}$
- $({}^{*}e - p) + t$
  $\tau = 7.51$
  $P = 2.98 \times 10^{-7}$
- $({}^{*}e - d) + d$
  $\tau = 9.45$
  $P = 6.22 \times 10^{-9}$

[D=2]

- $({}^{*}e - t) + p$
  $\tau = 2.41$
  $P = 8.03 \times 10^{-3}$
- $({}^{*}e - p) + t$
  $\tau = 4.11$
  $P = 2.72 \times 10^{-4}$
- $({}^{*}e - d) + d$
  $\tau = 7.88$
  $P = 1.42 \times 10^{-7}$

Magnetic monopole as the catalyzer of the nuclear cold fusion

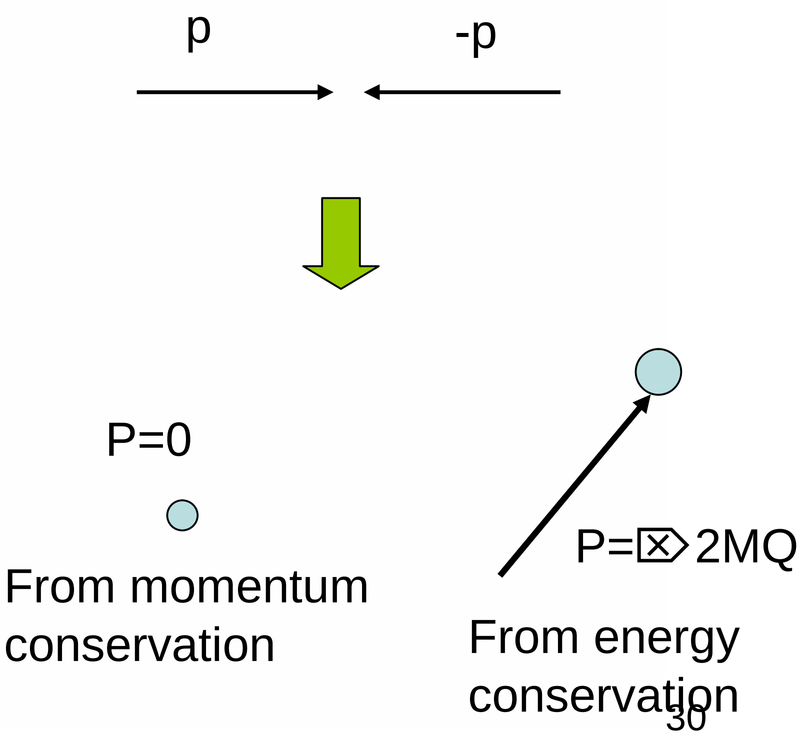
We saw that by the help of the magnetic monopole, the penetration factor P is increased to become the reasonable values. The magnetic monopole plays the roll as the catalyzer beautifully by lowering the repulsive barrier. However in general catalyzer is required to have other properties that it attract fuel particles and repels the product particle, and the latter property is necessary to avoid the inverse reaction to occur.

The monopole can discriminate the nuclei by the interaction with their magnetic moment. Therefore it does not attract ^4He. This is the novel property of the magnetic monopole. No other particle has such a property. For example, a particle with large negative charge can attract the fuel nuclei, however it attract ^4He also.

^4He dominance in the d+d reaction of the cold fusion (1)

It is amazing to see the differences of the final states of the d+d reaction for the case of the ordinary nuclear physics on one hand and for the case of the nuclear cold fusion on the other hand. In vacuum, because of the conservations of the energy and the momentum, the reaction of two-body $\rightarrow$ one-body is strictly forbidden when the reaction is the energy producing process, namely $Q > 0$.

The proof is simple.
If we choose the center of mass system of the initial state, the final state particle must have momentum $p_f = 0$ from the momentum conservation, on the other hand the energy conservation implies $p_f = \sqrt{2MQ}$. These results are in contradiction if $Q > 0$. Therefore two-body $\rightarrow$ one-body such as $d + d \rightarrow^4 He$ cannot occur. Instead, the two-body states such as $p + t$, $n +^3 He$ and $^4He + \gamma$ must occur in the final state. This is what we observe in the ordinary low energy nuclear physics.



^4He dominance in the d+d reaction of the cold fusion (2)

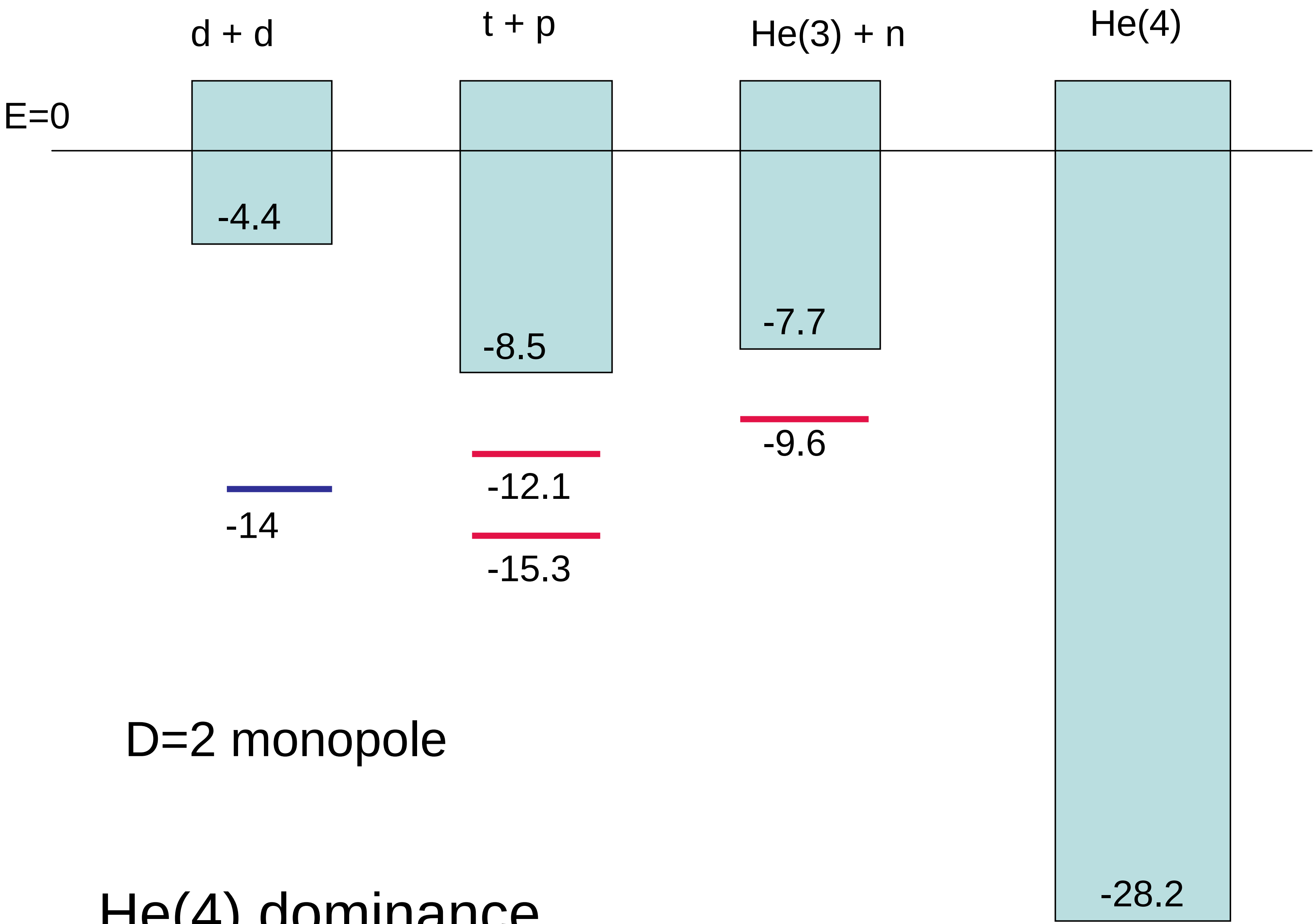
However the proof above breaks down if the reaction proceeds under the influence of the external potential, because it can absorb the momentum transfer $\Delta q = p_f = \sqrt{2MQ}$. Since for $d+d \to^4 He$, $Q = 23.97$ MeV. and M is the mass of ^4He, so the momentum transfer becomes $\Delta q = 422.9$ MeV./c $= 3.03\mu_\pi$. In order to absorb such large momentum transfer, the external potential must be "stiff" and "solid", also it must be localized. The size of the potential is estimated from the uncertainty principle $\Delta q \cdot \Delta r \sim 1$, and it turns out $\Delta r = 0.47$fm.. Therefore existence of the potential, whose size is order of fm. and the strength is large, is essential for the reaction $d + d \to^4 He$ to occur. Considering the radius of the external potential, the structure of the nano-meter scale of the background lattice is irrelevant to the cold nuclear fusion.

^4He dominance in the d+d reaction of the cold fusion (3)

Even if we start from $|d, d,^* e>$ bound state, because of the strong interaction, it oscillates among $|t, p,^* e>$, $|^3He, n,^* e>$ and $|^4He,^* e>$. Next graph is the energy spectra of these four states, the thresholds of the contineuos specta are E = -4.4, -8.5, -7.7 and -28.2 MeV. respectively, in which we used the values of the binding energy B = 2.2, 8.5, 7.7, and 28.2 MeV. respectively for d, t, ^3He and ^4He . Since the binding energies of the deuteron and the monopole are around B' = 1.7 and 4.8 MeV. respectively for $D = 1$ and $D = 2$ monopole. For $D = 2$, the starting energy is $E_{start} = -14$MeV., only the channel ^4He stays open. On the other hand $D = 1$ case is more delicate, if we adopt the value of the starting energy as $E_{start} = -7.8$MeV., the channels of $(t + p)$ and ^4He stay open, although because of the phase volume, the dominance of ^4He channel is still true. However small change of the starting energy E_{start} may open the $(^3He + n)$ channel. Therefore precise determination of the binding energy of the deuteron and monopole is inevitable for making the definite statement of the open channel.

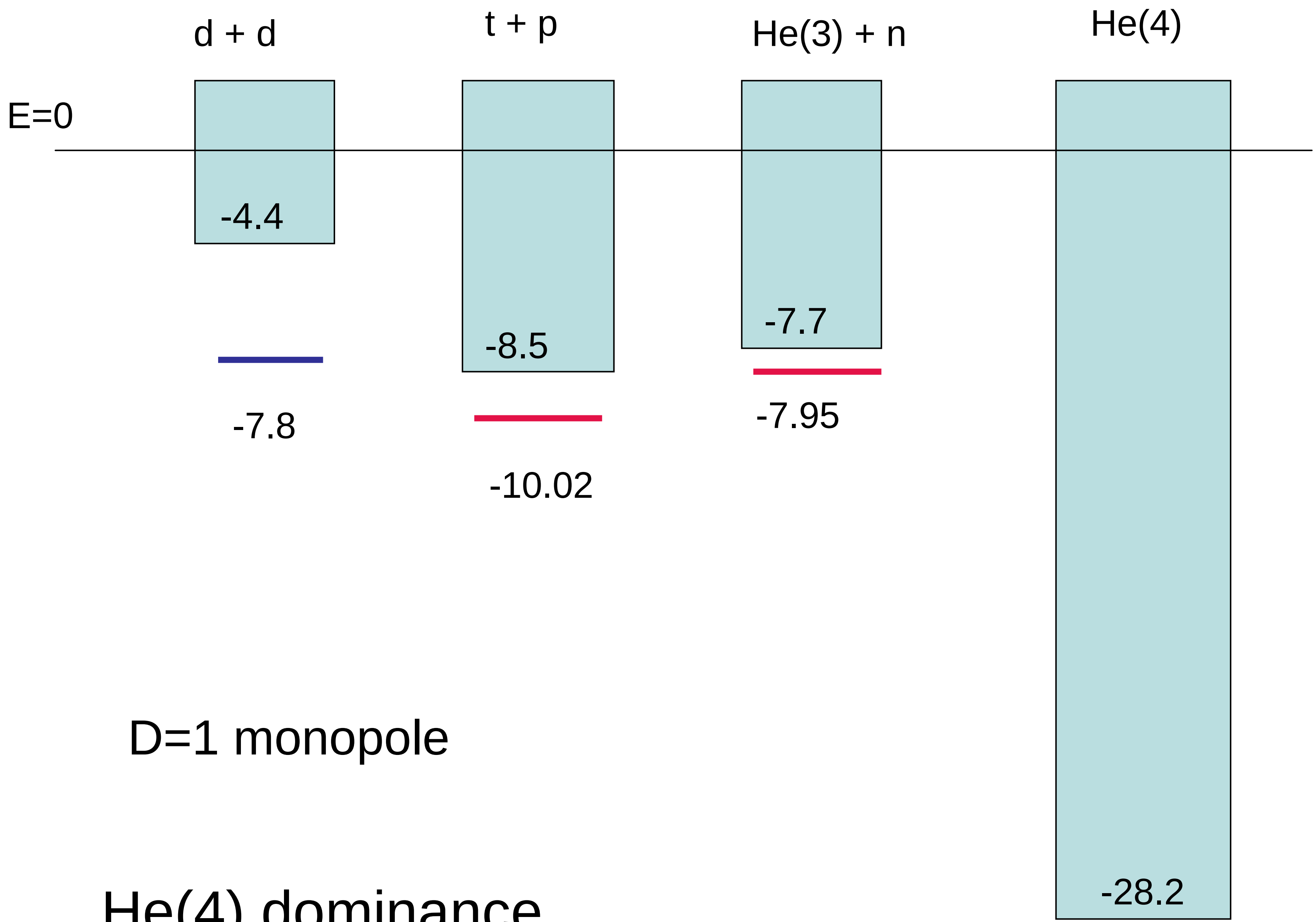
For the case of the low energy nuclear physics, the starting energy is $E_{start} = -4.4$MeV., and the open channels are $(t + p)$, $^3He + n$ and ^4He. However if the external potential does not exist, then the ^4He channel must be closed, so there remain $(t + p)$ and $^3He + n$ channels, as we know in the nuclear physics.

This "great reversal" of the final state is the most spectacular phenomenon in the cold fusion along with the occurance of the zero-incident energy nuclear reaction.



D=2 monopole

He(4) dominance



D=1 monopole

He(4) dominance